Theoretical and experimental investigation of flexural wave propagation in straight beams with periodic structures: Application to a vibration isolation structure

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A periodic binary straight beam with different cross sections is constructed and studied. The band structures of flexural waves in the structure are calculated with the plane-wave expansion method and the vibration attenuation spectra of a finite sample of it are calculated with the finite element method. Vibration experiment is performed in order to validate all the theoretical results, and the results match mainly. Finally, a vibration isolation structure is designed by using the feature of flexural wave band gaps of the beam with periodic structure, which provides an attenuation of about 30 dB in the frequency range of the band gaps. © 2005 American Institute of Physics. [DOI: 10.1063/1.1922068]

I. INTRODUCTION

The propagation of elastic wave in periodic composite called phononic crystals (PCs) has received a great deal of attention.1–16 Particular interests are focused on the existence of the so-called phononic band gaps (PBG) in which elastic waves are all forbidden. The study on PBG materials and structures is driven partly by the potential applications such as elastic filters, vibrationless environments for high-precision systems, transducer improvements, as well as pure physical concerns with the Anderson localization of sound and vibration. Only a few works about the applications of PCs have been published,14–16 but none of them has a direct application in vibration isolation or attenuation.

In this paper, we present a structure with the idea of PCs that can be used to isolate vibrations. Above all, a periodic binary straight beam with different cross sections is constructed. The band structures of flexural waves in the beam of infinite period are calculated with the finite element method. Vibration experiment is performed in order to validate all the theoretical results, and the results match mainly. Finally, a vibration isolation structure is designed by using the feature of flexural wave band gaps of the beam with periodic structure, which provides an attenuation of about 30 dB in the frequency range of the band gaps.

II. THEORY

Figure 1 illustrates a periodic binary straight beam with different cross sections. Aluminum and Lucite segments with different sizes of cross sections are arrayed along the x direction periodically in the beam. The lattice constant is a, and the cross section of the beam is a \(b_{ai} \times h_{ai}\) and \(b_{la} \times h_{la}\) rectangles, respectively. We consider the flexural wave in the x-y plane.

If the thickness \(h\) in the y direction and width \(b\) in the z direction in each segment of the beam are much smaller than the lattice constant \(a\) (often \(a \gg 5b\) and \(a \gg 5h\)), each unit cell can be regarded as an Euler–Bernoulli beam, where both the shearing deformation and rotational inertia of the cross sections are negligible. The flexural elastic wave equation along the x direction is

\[
\frac{\partial^2}{\partial x^2} \left[ E(x) I(x) \frac{\partial^2 y(x,t)}{\partial x^2} \right] + \rho(x) A(x) \frac{\partial^2 y(x,t)}{\partial t^2} = 0, \tag{1}
\]

where \(\rho(x)\) is the mass density, \(E(x)\) is the Young’s module, \(y(x,t)\) is the displacement in the y direction, \(I(x)\) is the moment of inertia, and \(A(x)\) is the cross-section area. The I and \(A\) are calculated with the size and shape of the cross section, and independent with the materials. For example, if the cross section is a \(b \times h\) rectangle, they can be calculated with \(I = bh^3/12, A = bh\).

For a periodic structure, Bloch’s theorem16 asserts that \(y(x,t)\) in Eq. (1) can be written as

\[
y(x,t) = \sum_{n} c_n \phi_n(x) e^{i \omega_n t},
\]

where \(\phi_n(x)\) is the Bloch function, \(c_n\) is the complex coefficient, and \(\omega_n\) is the frequency of the nth band.

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where $k$ is restricted within the irreducible Brillouin zone and $y_{k}(x)$ is a function with the same period as $1/\rho(x)$, $E(x)$, $A(x)$, and $I(x)$ which can be expanded in a Fourier series
\[
y(x,t) = \sum_{G_0} e^{i(kx-\omega t)}y_{k}(x),
\]
where $f(x)$ stands for $1/\rho(x)$, $E(x)$, $A(x)$, and $I(x)$, respectively, and $G$ is the reciprocal lattice vector.

Substituting Eqs. (2) and (3) into Eq. (1), we obtain
\[
\omega^2 \sum_{G_0} \left( \frac{G' - G_0}{2} \right) A \left( \frac{G' - G_0}{2} \right) y_{k}(G_0) = \sum_{G_0} (G_0 + k)^2(G' + k)^2E \left( \frac{G' - G_0}{2} \right) I \left( \frac{G' - G_0}{2} \right) y_{k}(G_0),
\]
where $G' = G_0 - G$.

Equation (4) is an infinite set of linear equations. In practice, only a finite number of vectors $G'$ and $G_0$ (plane waves) can be employed in the calculation. We employ 441 plane waves in this paper and the convergence is satisfied. The band structure represents all the stable flexural wave modes propagating in the infinite periodic structure of the binary straight beam with different cross sections.

As for a finite periodic beam structure, the attenuation (unstable) modes in the band gaps can exist. Thus, the propagation of flexural waves within the specific frequency ranges can only be weakened, not totally forbidden. In order to describe the propagation of flexural wave in the finite sample correctly, we employ the FEM to calculate the vibration attenuation spectra of it.

III. THEORETICAL AND EXPERIMENTAL RESULTS

Using the PWE in this paper, we calculate the flexural wave band structure of the beam with infinite periods illustrated in Fig. 1. The structural parameters employed in the calculations are $a=0.07$ m, filling fraction $f=2.5$ (where $f=l_{AI}/l_{Lu}$, $l_{AI}$ and $l_{Lu}$ are the lengths of Al and Lucite segments), $b_{AI}=b_{Lu}=0.01$ m, and $h_{AI}=0.005$ m. The material parameters used are $\rho_{AI}=2799$ kg m$^{-3}$, $E_{AI}=7.2 \times 10^{10}$ Pa for aluminum; and $\rho_{Lu}=1142$ kg m$^{-3}$, $E_{Lu}=2.01 \times 10^{10}$ Pa for Lucite. Figure 3(a) shows the calculated flexural wave band structures. With the FEM, we calculated the vibration attenuation spectra of a finite sample of the beam with six and 10 periods, which are shown in Fig. 3(b), respectively.

In order to verify the results calculated with the PWE and the FEM, a vibration experiment is performed. The experimental system is shown in Fig. 2. Here, a white-noise signal with a bandwidth from 0 to 3.2 kHz is input into the vibration shaker, which transmits vibrations to the left end of the beam through the force transducer. Then the flexural waves propagate through the beam. The acceleration at the right end of the beam is measured with an accelerometer. The measured results are shown in Fig. 3(b) comparing with that calculated with the FEM.

In Fig. 3(a), the lowest gap in the dispersion curves [solid lines in Fig. 3(a)] of the flexural wave locates between 501 and 1430 Hz. The frequency range of the large attenuation in the calculated vibration attenuation spectra (the dotted and the dash-dotted lines) is from 480 to 1350 Hz. The frequency range of the large attenuation in the measured vibration attenuation spectra (the solid and dashed lines) is from 430 to 1100 Hz. All the theoretical and experimental results match in the main. The measured and calculated results also show that the attenuation in the band gap is in the direct ratio with the periodic number.

The influence of the rotational inertia of the cross section and the shear deformation is involved in the FEM and experiment results, while it is not considered in the PWE method. Because both the shear deformation and the rota-
tional inertia tend to lower the value of the frequencies, the start and stop frequencies of the sharp drops are little smaller than those of the flexural wave band gaps.

IV. APPLICATION TO A VIBRATION ISOLATION STRUCTURE

A vibration isolation structure illustrated in Fig. 4 is designed with the band-gap feature of the flexural waves in the beam with periodic structure. Aluminum and Lucite bars with different rectangular cross sections are jointed together and arrayed alternatively in a framework of six layers. Each side of the layers in square shape consists of two symmetrically connected periods of the periodic straight beam illustrated in Fig. 1. Four Al blocks located at the four apaxes (or at the middle of four sides) of each layer act as a shoring connecting the layers one by one. The shoring blocks are made as short as possible in order to ensure that the displacements can transfer through them without obvious delay or loss, and the mass of the block is small enough compared with other parts of the structure. The bearing plate is fixed on the top layer. When the vertical vibration occurs, it is transferred to four apaxes of the bottom layer with same phase through the shoring, and the flexural waves in eight periods of the beam are excited synchronously. Whereafter, the vertical vibrations on each end of the eight periods of the beam is transferred to the second layer. Similar process occurs until the vibration propagates through the whole structure in the form of flexural wave and transferred to the bearing plate. The correspondence between the wave propagation behaviors in structures illustrated in Figs. 1 and 4 is not totally straightforward. However, with the foregoing analysis, the behavior of the elastic waves propagating in each route (as illustrated with arrows in Fig. 4) through the structure is similar with that of the flexural waves in a periodic straight beam with six periods. The lattice constant \(a\) is about 0.07 m, and the filling fraction \(f\) is about 2.5 (for the existence of the shoring, we cannot get the exact values). The cylindrical load of aluminum attached to the bearing plate is 1.5 kg. In the experiment, one accelerometer is adhered to the shoring, and the other is on the top of the load.

In Fig. 5, the frequency range of the large attenuation in the measured vibration attenuation spectra of the vibration isolation structure is from 350 to 1250 Hz, which match with the theoretical and experimental results of the periodic binary straight beam mainly. The measured results show that the attenuations in the band gap are in direct ratio with the period number.

V. CONCLUSION

In conclusion, the flexural wave in a periodic binary straight beam with different cross sections is studied theoretically with the PWE and FE methods and experimentally with a vibration experiment. The calculated and measured results show that the existence of the band gaps match mainly. Finally, a vibration isolation structure is designed by using the feature of flexural wave band gaps of the beam with periodic structure, which provides an attenuation of about 30 dB in the frequency range of the band gaps.

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