Propagation of steady-state vibration in periodic pipes conveying fluid on elastic foundations with external moving loads

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**A R T I C L E   I N F O**

Article history:
Received 20 June 2012
Received in revised form 7 September 2012
Accepted 27 September 2012
Available online 4 October 2012
Communicated by R. Wu

Keywords:
Pipe
Band gap
Moving load
Elastic foundation

**A B S T R A C T**

The propagation of steady-state vibration in a periodic pipe conveying fluid on elastic foundation with an external moving load is studied using wave propagation and attenuation theory. Wavenumbers and propagation properties in a moving coordinate system are investigated. The propagation constants are calculated using transfer matrix theory to determine whether the perturbation, which is introduced by an external moving load, can propagate through the pipe or not. The Bragg and locally resonant band gaps, corresponding to the velocity field, can exist in a periodic pipe system. In addition, the effects on both types of band gaps have been analysed.

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1. Background

Moving loads exist in many elastic structures, including pipe systems conveying fluid, and bridges, rails. The analysis of an elastic structure with external moving loads has triggered a great deal of attention due to its unusual physical characteristics [1–3]. In practical terms, the moving loads are affected by any change in the elastic foundation. Kenney investigated the stability of the steady-state response of a beam on an elastic foundation by formulating the beam’s equation of motion with respect to a reference coordinate system with the moving load [1]. Additionally, Rafioyannis pointed out that it is difficult to achieve the critical speeds for a beam on elastic foundation, since they are significantly higher than the ones of the corresponding free single-span simply supported beam [4]. Recently, piping systems conveying fluid with moving loads have been investigated. Yoon studied the effects of an open crack and a moving mass on a simply supported pipe conveying fluid [5]. A study of the dynamic behaviour of a pipe conveying fluid with a spring mass moving was also performed by Morteza [6].

The study of wave propagation in periodic structures has a long history [7]. Interest in this phenomenon was sparked by the existence of complete elastic band gaps, within which the propagation of sound and vibration are forbidden. There are two types of band gap formation mechanisms: the Bragg scattering mechanism [8] and the locally resonant (LR) mechanism [9]. Most studies are focused on the band gap formation mechanism, calculation methods, as well as the application of band gap in engineering materials and structures [10–12]. Some authors have investigated how to use the band gap to control the propagation of vibration in a pipe system conveying fluid [13–15].

Research on the dynamic stability of periodic structures with moving loads is scant [16,17]. An important study that is relevant to this work was performed by Ruzzene and Baz [18]. They extended Kenney's method, described in Ref. [1], to investigate the steady-state response of axi-symmetric shells with periodic stiffeners. Significantly, this study concluded that the periodic stiffening rings along the shell can improve the dynamic stability characteristics.

In this Letter, the propagation of steady-state vibration in a periodic pipe on an elastic foundation and with external moving loads is studied. The steady-state analysis method is introduced into the periodic pipe system while considering the two band gap formation mechanisms. Hence, two types of band gaps, both corresponding to the velocity field, are obtained. Finally, various effects are analysed at the end of this Letter.

2. Wave propagation characteristics in a uniform pipe on elastic foundation with moving loads

For an infinitely long pipe on an elastic foundation with moving loads, as shown in Fig. 1(a), neglecting the gravitational forces, internal damping, externally imposed tension and pressurisation...
effects, the well-known governing equation of flexural vibration with free boundary condition is given by [19,20]
\[
EI \frac{\partial^4 w}{\partial x^4} + m_f u^2 \frac{\partial^2 w}{\partial x^2} + 2m_f u \frac{\partial^2 w}{\partial x \partial t} + (m_f + m_p) \frac{\partial^2 w}{\partial t^2} + k_f w = F_0 \delta(x - vt),
\]
where \( w \) is the flexural displacement; \( EI \) is the flexural rigidity of the pipe; \( m_f \), \( m_p \) and \( t \) are the fluid and pipe masses per unit length, and time. \( F_0 \delta(x - vt) \) is the external load travelling at constant velocity \( v \) along the pipe length. \( F_0 \) is the magnitude of the load, and the stiffness coefficient of the foundation is \( k_f \).

The cross-section of the pipe is illustrated in Fig. 1(b). For comparison, the Bragg pipe with a periodically alternating composite material and the LR pipe with periodic oscillators are presented in Figs. 1(c) and (d), respectively.

Utilising a moving coordinate system \( \xi [1,2,18] \)
\[ \xi = x - vt. \]

Then
\[ \tilde{w}(\xi) = w(x - vt). \]

For the calculation of the steady-state response of the pipe, a new coordinate system is defined, and all of the partial derivatives with respect to time can be set to zero [1,2,18]. Thus, Eq. (1) can be rewritten as
\[
EI \frac{\partial^4 \tilde{w}}{\partial \xi^4} + (m_f u^2 - 2m_f uv + (m_f + m_p)v^2) \frac{\partial^2 \tilde{w}}{\partial \xi^2} + k_f \tilde{w} = F_0 \delta(\xi). \quad (2)
\]

For a harmonic travelling wave \( \tilde{w}(\xi) = Ae^{i\xi} \), one can find the dispersion relations corresponding to Eq. (2)
\[
Elk^4 + (m_f u^2 - 2m_f uv + (m_f + m_p)v^2)k^2 + k_f = 0. \quad (3)
\]

The solution of Eq. (3) may be expressed as
\[
k_f^2 = -\frac{m_f u^2 - 2m_f uv + (m_f + m_p) v^2}{2EI} + \frac{1}{2EI} \sqrt{(m_f u^2 - 2m_f uv + (m_f + m_p)v^2)^2 - 4Elk_f^2},
\]
For a given material and geometric parameters of the pipe, the wavenumber \( k \) is a function of fluid velocity \( u \) and load velocity \( v \). The harmonic solution of Eq. (2) has the following form
\[
\tilde{w}(\xi) = W_1 e^{-k_1 \xi} + W_2 e^{-k_2 \xi} + W_3 e^{k_1 \xi} + W_4 e^{k_2 \xi},
\]
where the coefficients \( W_1, W_2, W_3, \) and \( W_4 \) can be determined by the boundary conditions.

Waves are free to propagate when the real parts of the associated wavenumbers are equal to zero [18]. To determine when the real parts of the associated wavenumbers are equal to zero, the discriminant of Eq. (2) can be set to zero. Thus, a relationship between variables \( u \) and \( v \) can be obtained as follows
\[
(m_f u^2 - 2m_f uv + (m_f + m_p)v^2)^2 - 4Elk_f = 0. \quad (4)
\]

Eq. (5) is the general equation for an ellipse in variables \( u \) and \( v \), and the area can be calculated directly by
\[
S = 4\pi \left( \frac{Elk_f}{3m_f^2 + 4m_f m_p} \right)^{1/2}. \quad (6)
\]

In the calculation, the material of the pipe is aluminium, whose elastic parameters are \( \rho_{Al} = 2730 \text{ kg/m}^3 \), \( E_{Al} = 7.756 \times 10^10 \text{ Pa} \). The inner and outer radii of the pipe are \( r_1 = 0.045 \text{ m} \) and \( r_2 = 0.05 \text{ m} \), respectively. The stiffness for the elastic foundation is \( k_f = 1 \times 10^{10} \text{ Pa} \). The fluid in the pipe is water, with a density of \( \rho_w = 1000 \text{ kg/m}^3 \).

The elliptic curve in Fig. 2 describes the relationship between fluid velocity \( u \) and moving load velocity \( v \) in Eq. (5). Outside the elliptic area, the real parts of the wavenumbers are equal to zero, and the waves are free to propagate; inside the elliptic area, the real parts of the wavenumbers are not equal to zero, and the waves cannot propagate. For instance, when \( u = 100 \text{ m/s} \), we can obtain a critical moving load velocity \( v_0 = 320 \text{ m/s} \), i.e., \( v > v_0 \), real(\( k \)) = 0. This result is demonstrated in Fig. 3(a), in which the relationship between wavenumbers \( k \) and moving load velocity \( v \) is illustrated. For \( u = 400 \text{ m/s} \), we could obtain two critical moving load velocities, \( v_1 = 67 \text{ m/s} \) and \( v_2 = 423 \text{ m/s} \), i.e., \( v < v_1 \) or \( v > v_2 \), real(\( k \)) = 0, as demonstrated in Fig. 3(b). This finding indicates that for the ranges of \( v < v_0 \) or \( v_1 < v < v_2 \), the perturbation induced by the moving load cannot propagate along the pipe length in the moving coordinate system. Therefore, if we adjust the fluid velocity \( u \) (or moving load velocity \( v \)) the perturbation introduced by the moving load can be reduced, and the stability of the pipe can be improved.

The effects of the foundation stiffness \( k_f \) and pipe material properties on the elliptic curve are presented in Figs. 4(a) and (b).
The material parameters employed in the calculation are Epoxy, aluminium and steel, respectively. These data are in good agreement with the dashed, solid and dash–dot lines correspond to epoxy, aluminium and steel, respectively. These data are in good agreement with the foundation stiffness is increased. At the same time, for different pipe materials, a higher the material modulus results in a larger the elliptic area, as illustrated in Fig. 4(b), wherein the size of the pipe is increased.

The relationship between wavenumbers and moving load velocity \( v \) \( u = 100 \) m/s and \( u = 400 \) m/s.

Fig. 3. The relationship between wavenumbers \( k_f \) and moving load velocity \( v \) (a) \( u = 100 \) m/s and (b) \( u = 400 \) m/s.

Fig. 4. The effect on the elliptic curve from (a) elastic foundation stiffness \( k_f \) and (b) the pipe material properties.

respectively. Fig. 4(a) presents three lines: a dashed line, a continuous line, and a dash–dot line, corresponding to the foundation stiffness \( k_f = 1 \times 10^6 \) Pa, \( k_f = 1 \times 10^7 \) Pa and \( k_f = 1 \times 10^8 \) Pa, respectively. Fig. 4(a) shows that the elliptic area is enlarged as the foundation stiffness is increased. At the same time, for different pipe materials, a higher the material modulus results in a larger the elliptic area, as illustrated in Fig. 4(b), wherein the size of the pipe is increased.

3. A Bragg periodic pipe on elastic foundation with moving loads

A periodic pipe system comprised of two materials is presented in Fig. 1(c). The system consists of an infinitely long, alternating repetition of pipe material \( A \), with length \( a_1 \), and material \( B \), with length \( a_2 \). Thus, the lattice constant is \( a = a_1 + a_2 \).

For a periodic pipe, the continuity of displacement \( \tilde{w} \), slope \( \tilde{w}' \), bending moment \( E I \tilde{w}' \) and shear force \( E I \tilde{w}' \) at the interfaces between pipe \( A \) and pipe \( B \) in cell \( n \) (i.e., \( x = na + a_1 \)) yield

\[
\begin{align*}
\tilde{w}_{n,A} & = \tilde{w}_{n,B}, \\
\tilde{w}'_{n,A} & = \tilde{w}'_{n,B}, \\
E A I A \tilde{w}'_{n,A} & = E B I B \tilde{w}'_{n,B}, \\
E A I A \tilde{w}''_{n,A} & = E B I B \tilde{w}''_{n,B}.
\end{align*}
\]

The matrix form of Eq. (7) can be written as

\[
K_1 \mathbf{W}_{n,A} = H_1 \mathbf{W}_{n,B},
\]

where \( \mathbf{W} = [W_1, W_2, W_3, W_4]^T \).

Analogously, the continuity at the interfaces between the \((n-1)\)th cell and \(n\)th cell (i.e., \(x = na\)) yields

\[
K W_{n,A} = H W_{n-1,B}.
\]

The relationship between the \(n\)th cell and \((n-1)\)th cell is given by

\[
\mathbf{W}_{n,B} = \mathbf{T}_b \mathbf{W}_{n-1,B},
\]

where \( \mathbf{T}_b = H^{-1}_1 K_1 K^{-1} \) is the transfer matrix of Bragg periodic pipe.

Due to the periodicity of the infinite structure in the \(x\) direction, the vector \( \mathbf{W}_u \) must satisfy the Bloch theorem [21]

\[
\mathbf{W}_{n,B} = e^{i\mu x} \mathbf{W}_{n-1,B},
\]

where \( \mu \) is referred to as the propagation constant, with the real part denoting the attenuation constant (decay of the amplitude of a wave propagating from one cell to the following) and the imaginary part denoting the corresponding phase constant [18].

It follows that the eigenvalues of the infinite periodic pipe are the roots of the determinant, i.e.,

\[
|\mathbf{T}_b - e^{i\mu I}| = 0,
\]

where \( I \) is a \(4 \times 4\) unit matrix. For a given fluid velocity \( u \), Eq. (12) gives the values of propagation constant \( \mu \). Therefore, wave propagation is possible within some load velocity ranges, where \( \mu \) is pure imaginary number (pass bands), while attenuation occurs for
The band gap width increases monotonically with the fluid velocity and the ending edge of band gap by inverted triangles, the fluid velocity values that result in the real part of the propagation constant being nonzero (stop bands).

As an example, we choose material A as epoxy, material B as aluminium, and the lattice constant as $a = 1$ m, wherein $a_1 = 0.2$ m; the other parameters are kept the same as in Fig. 2. The real parts of the propagation constants of the periodic pipe for fluid velocity $u = 50$ m/s are presented in Fig. 5(a). There are two ranges where the real parts of the propagation constants are not equal to zero. The first attenuation range is 0–212 m/s. This is due to the effects of the elastic foundation, not the causation of the periodic structure. The critical velocity value, 212 m/s, for the periodic pipe is the critical velocity value for uniform epoxy and that for uniform aluminium pipes. The second attenuation range is 412–484 m/s. This attenuation zone is due to material imperfection effects of the periodic structure. While the term “band gap” is defined in a periodic structure corresponding to the frequency band, the attenuation areas in Fig. 5(a) can be also termed “band gap”, except that here, it is in the velocity field. Within the velocity band gap, the excitation is localised to the moving load, whereas in other ranges, the perturbation can propagate along the length of the pipe system freely. For $u = 300$ m/s, the real parts of the propagation constants of the periodic pipe are illustrated in Fig. 5(b). As the fluid velocity $u$ varies, the propagation constants vary, and the band gap range moves to higher load velocity $v$ zone. The two attenuation zones are 110–350 m/s and 536–652 m/s. In this case, the perturbation can propagate freely through the pipe in a low velocity range ($< 110$ m/s). This finding is in agreement with the wavenumber properties shown in Fig. 3(b).

Furthermore, we investigate the effects of fluid velocity $u$ and foundation stiffness $k_f$ on the band gap while the other parameters are kept the same as in Fig. 5(a). As shown in Fig. 6, in which the beginning edge of band gap is denoted by triangles pointing up and the ending edge of band gap by inverted triangles, the fluid velocity $u$ is changed monotonically from 10 m/s to 400 m/s. The results in Fig. 6(a) show that the first band gap edge velocities and the band gap width increases monotonically with the fluid velocity. As shown in Fig. 6(b), as the stiffness of elastic foundation, $k_f$, is increased from $1 \times 10^4$ Pa to $1 \times 10^6$ Pa, the band gap edge velocities decrease, but the band gap width is enlarged.

It is well known that the length ratio (i.e., filling ratio) plays an important role in the dynamics of one-dimensional periodic structures, and the band gap width will reach a maximum at some intermediate length ratio. However, this is not the case for a periodic pipe with a moving load. Fig. 6(c) shows the beginning velocities and ending velocities of the first band gap versus the length ratio $a_1/a$. As the length ratio, $a_1/a$, increases, the first band gap edge velocities and band gap width decreases, and they will even disappear when a certain ratio is reached. However, another band gap emerges at a larger length ratio.

4. An LR periodic pipe on elastic foundation with moving loads

Fig. 1(d) shows an LR periodic pipe with its harmonic oscillators attached periodically. The LR oscillator consists of a spring with spring constant of $K$ and a mass $M$.

Considering the equilibrium condition for the forces in the $y$-axis, one obtains [22,23]

$$f_n(t) + m\ddot{Z}_n(t) = 0,$$

where $f_n(t)$ are the interactive forces between the spring-mass system and the pipe at the attaching points $x_n$. $Z_n(t) = V_n \exp(i\omega t)$ is the displacement of the nth LR oscillator at the centre of gravity, and $V_n$ is the amplitude of displacement. The force $f_n(t)$ is given by

$$f_n(t) = K[Z_n(t) - w(x,t)] = K[V_n - w(x)] \exp(i\omega t) \triangleq F_n \exp(i\omega t).$$

Substituting Eq. (14) into Eq. (13) and in the moving coordinate system $x$, one can obtain

$$V_n = \frac{K\ddot{w}}{K - M\dot{v}^2}. \quad (15)$$

And substituting Eq. (15) into Eq. (14) yields the following

$$F_n = \frac{KM\dot{v}^2}{M\dot{v}^2 - \ddot{w}}. \quad (16)$$

The continuity of displacement, slope, bending moment and shear force at attaching point $x_n = na$ yield

$$\ddot{w}_n = \ddot{w}_{n-1}, \quad (17a)$$

$$\ddot{w}_n' = \ddot{w}_{n-1}', \quad (17b)$$

$$EI\dddot{w}_n'' = EI\dddot{w}_{n-1}'', \quad (17c)$$

$$EI\dddot{w}_n'' + F_n = EI\dddot{w}_{n-1}''. \quad (17d)$$

Based on Eqs. (17), the relationship between the nth cell and $(n-1)$th cell is given by

$$\mathbf{W}_n = T_1 \mathbf{W}_{n-1}, \quad (18)$$

where $T_1$ is the transfer matrix for LR periodic pipe.

Analogous to Eqs. (11) and (12) in Bragg periodic pipe, the LR periodic pipe propagation constant $\kappa$ in LR periodic pipe can be calculated.

Fig. 7(a) shows the real parts of propagation constants in an LR periodic pipe with fluid velocity $u = 50$ m/s. The values used in the calculation are $a = 0.5$ m, $K = 1 \times 10^5$ Pa, $M = 0.7$ kg and $k_f = 1 \times 10^6$ Pa, and the material for the pipe is aluminium.
can find that an LR band gap exists in velocity field, due to local resonance, and it appears in the 341–378 m/s velocity range. The attenuation at beginning edge of the gap is the minimum, and becomes stronger as the velocity increases, finally reaching the maximum at its ending edge. According to Eq. (15), the resonant velocity is $v_0 = \sqrt{M/K}$, which means the resonant velocity of the oscillator is located at the ending edge. This is the opposite of the attenuation behaviour for the LR gap in frequency field, where the resonant frequency of oscillator is located at beginning edge and the attenuation becomes weaker as the frequency increases. Compared with the Bragg band gap, the LR band gap range in velocity field is narrower. This is also true for the LR band gap in frequency field. Additionally, the LR band gap velocity depends only on the resonant velocity $v_0$, whereas the fluid velocity and the elastic foundation stiffness do not have an effect. Fig. 7(b) shows the dependence of the real parts of the propagation constants on the fluid velocity $u = 300$ m/s. In this case, the attenuation range induced by the elastic foundation is strongly correlated with the range of the LR band gap. Above the resonant velocity, $v_0$, the effect of the elastic foundation disappears, and the steady-state vibration can propagate freely throughout the pipe.

5. Conclusion

In conclusion, using wave propagation and attenuation criteria, the propagation of steady-state vibration through a periodic pipe
on an elastic foundation with external moving loads has been studied and explained in this Letter.

After introducing a moving coordinate system, the wavenumbers and travelling wave characteristics are studied. By adjusting the fluid velocity $u$ or moving load velocity $v$, the perturbation in the pipe system introduced by the moving load can be attenuated. We assert that this attenuation range is determined by the elastic foundation, not the band gap.

Using the transfer matrix theory, the propagation constants of a periodic pipe system are calculated. Band gaps, corresponding to the velocity field, can exist in a periodic pipe system and are caused by two different mechanisms. The effects of fluid velocity, elastic foundation stiffness and material length ratio on the Bragg band gap are clear. However, for the LR band gap, the only factor that elicits an effect is the LR oscillator. These band gap properties in the velocity field are quite different from those in the frequency field.

These findings will be helpful for improving the stability of the pipe system conveying fluid.

Acknowledgements

The authors are grateful for the support provided by the National Natural Science Foundation of China (Grant Nos. 10902123 and 51075392).

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