Analysis of absorption performances of anechoic layers with steel plate backing

Hao Meng, Jihong Wen, Honggang Zhao, Linmei Lv, and Xisen Wen

Key Lab of Photonic/Phononic Crystals of Ministry of Education, and Laboratory of Science and Technology on Integrated Logistics Support, National University of Defense Technology, Changsha, Hunan 410073, People’s Republic of China

(Received 12 November 2011; revised 16 May 2012; accepted 22 May 2012)

Rubber layers with air-filled cavities or local resonance scatters can be used as anechoic coatings. A lot of researches have focused on the absorption mechanism of the anechoic coatings. As the anechoic coatings are bonded to the hull of submarine, the vibration of the hull should not be neglected when the analysis of the absorption characters is carried out. Therefore, it is more reasonable to treat the anechoic coating and the backing as a whole when the acoustic performance is analyzed. Considering the effects of the steel plate backing, the sound absorption performances on different models of anechoic coatings are investigated in this paper. The Finite Element Method is used to illustrate the vibrational behaviors of the anechoic coatings under the steel backings by which the displacement contours is obtained for analysis. The theoretical results show that an absorption peak is induced by the resonance of the steel slab and rubber layer. To illuminate the effects of the steel slab backing on the acoustic absorption, the thicknesses of the steel slab and the anechoic layer are discussed. Finally, an experiment is performed and the results show a good agreement with the theoretical analysis.

© 2012 Acoustical Society of America. [http://dx.doi.org/10.1121/1.4728198]

PACS number(s): 43.30.Ky, 43.20.Fn [NX] Pages: 69–75

I. INTRODUCTION

To prevent detection by active sonar, submarines are often equipped with an anechoic coating. In the past few decades, the absorption mechanisms of such layered configurations have been extensively investigated. Until now, many studies have investigated the design of rubber coatings with different acoustic structures, such as air-filled cavities, microspheres, or locally resonant phononic crystals. Oberst and Mayer were the first to study anechoic coatings containing spherical air cavities. Oberst quantified the acoustic performance of such anechoic coatings using a lumped system approximation, while Meyer calculated the resonance frequency of the bubble in an infinite solid. Theoretical and experimental investigations have showed the existence of two types of resonance mechanisms in such anechoic coatings, i.e., one mechanism is the radial motion of the cylinder ring and the other is the drum-like vibration of the cover layer. The vibration that plays a decisive role in sound absorption is determined by the relative stiffness of the cover layer and the perforated layer. Gaunaurd predicted the effective dynamic properties of elastic materials containing random distributions of fluid-filled spherical perforations. Considering the complexity of the acoustic structures embedded within the anechoic coating, it is difficult to obtain an analytical solution for their acoustic performances. Therefore, the Bloch theorem for periodic structures is used in combination with the Finite Element Method (FEM) to simplify the anechoic coating model; it is not restricted by the structure’s geometrical dimensions or material properties. Hennion and Easwaran investigated the reflection characteristics of anechoic coatings containing a double periodic elastic structure, and validated this model by physical experiments. Recently, the design of underwater noise control linings was studied using FEM, and the absorption properties of semi-infinite water-backed, visco-elastic materials containing locally resonant ellipsoidal scatterers were investigated.

In order to gain the absorption mechanism of the anechoic layers, the special backings, such as semi-infinite water or steel backings, are selected to eliminate their effects on the absorption performance. Nevertheless, the anechoic coatings are bonded to the hull of submarine, and the vibration of the hull should not be neglected when the analysis is performed. Therefore, it is more reasonable to treat the anechoic coating and the backing as a whole when the acoustic performance is investigated.

The sound absorption mechanisms of different anechoic coatings under the steel plate backings are discussed using the FEM in this paper. Consequently, the paper is organized as follows. Section II gives a brief review of the basic FEM, which will be used in the analysis of the vibration modes of the anechoic layers. With the steel plate backing, the sound absorption mechanisms of different anechoic coatings, such as the homogenous slab, the Alberich anechoic layer, and the local resonance slab, are investigated in detail in Sec. III. To illuminate the effects of the steel slab backing on the acoustic absorption, the thicknesses of the steel slab and the anechoic layer are discussed in Sec. IV. And an experiment is carried out.

Author to whom correspondence should be addressed. Electronic mail: wenxs@vip.sina.com.
out to validate the theoretical results in this section. Finally, Sec. V contains a summary of the results.

II. OUTLINES OF FEM FOR ANALYSIS ON COUPLING OF FLUID AND STRUCTURE

The analysis on the underwater sound performances for the anechoic coatings belongs to the acoustic problems of the coupling of fluid and structure. On one hand, the incident waves from the fluid evoke the vibration of the anechoic layers; on the other hand, the oscillation on the interface of the fluid and structure will affect the propagation of the waves in the water. By using the Galerkin method, the discrete equations of the FEM for the fluid-structure interaction problem can be written in the assembled form as

$$
\begin{align*}
\begin{bmatrix}
R^T & K^s - \omega^2 M + \text{i}\omega C^s \\
K^p - \omega^2 M^p + \text{i}\omega C^f & -\rho_0 \omega^2 R
\end{bmatrix}
\begin{bmatrix}
p \\
u
\end{bmatrix}
= 
\begin{bmatrix}
F^s \\
F^f
\end{bmatrix},
\end{align*}
$$

(1)

where \( p \) and \( u \) are the pressure and displacement vectors; \( K, M, \) and \( C \) are the structural stiffness, coherent mass and damping matrices, respectively; the superscripts \( s \) and \( p \) denote the solid and fluid components, respectively; \( R \) is a matrix that describes the coupling between the structure and the fluid and relates to the kinematic and dynamic interface continuity equations; \( \rho_0 \) is the fluid density; \( \omega \) denotes the angular frequency; \( F^s \) and \( F^f \) denote the nodal values of the applied forces in the structure and fluid, respectively.

For the viscoelastic material, its Young’s modulus can be expressed as a complex number

$$
\tilde{E} = E(1 + \text{i}\eta),
$$

(2)

where \( \eta \) is the loss factor, therefore, the damping matrix \( C^s \) can be expressed by the stiffness matrix \( K^s \) and the loss factor \( \eta \)

$$
C^s = \frac{\eta}{\omega} K^s.
$$

(3)

For the fluid, it is assumed to be inviscid, which means that its viscosity causes no dissipative effects. Hence, the damping matrix \( C^f \) for the inner of the fluid is equal to zero. To simulate the infinite flow field, the outer boundary of the fluid should be the no-reflecting.

The discrete expression for the fluid damping matrix \( C^f \) at the outer boundary follows that

$$
C^f = \frac{2}{c} \int_N N^T dS,
$$

(4)

where \( c \) is the sound speed in fluid, \( z \) is the absorption coefficient for the fluid outer boundary, and \( N \) is the shape functions for the sound pressure. In order to achieve a no-reflecting boundary, \( z \) should be equal to 1.

According the theory in the literature, the acoustic loads of the incident plane wave should be written into the discrete form using the shape functions and the incident pressure normal gradient

$$
F^f = \int_N \Phi \Omega dS',
$$

(5)

where \( \Phi \) is the incident pressure normal gradient.

Because the positioning of the acoustic structure is periodic, the anechoic coating can be treated as an extension of one cell in an infinite plane which is normal to the direction of the incident waves. Therefore, only a single cell including the fluid domains needs to be meshed, which is divided into nine parts including four corner lines \( C1, C2, C3, C4 \), four rectangle surfaces \( S1, S2, S3, S4 \), and the inner domain. According to the Bloch condition, any space function \( \chi \) (displacement, pressure,...) can be incorporated in the following form:

$$
\chi(x + d_x, y + d_y, z) = \chi(x, y, z) e^{\text{i}d_x \sin \theta \cos \phi} e^{\text{i}d_y \sin \phi}.
$$

(6)

The periodic boundary conditions are shown as the equations (6)–(7) in Ref. 12 in detail. What should be noted is that the force at the corner should satisfy the following equation:

$$
F_{m}^{c1} + F_{m}^{c2} e^{\text{i}\theta_{\phi}} + F_{m}^{c3} e^{\text{i}\phi} + F_{m}^{c4} e^{\text{i}(\theta_{\phi} + \phi_{\phi})} = 0,
$$

(7)

where \( F_m^{c} \) represents the force on the corner line \( C_i \).

By solving the above equations, one can obtain the displacement \( u \) for the structure and the pressure \( p \) in the fluid, and reflectance \( R \) and transmittance \( T \) can also be obtained. The absorption \( \alpha \) can then be calculated from the following equation:

$$
\alpha = 1 - R^2 - T^2.
$$

(8)

III. ANALYSIS ON SOUND ABSORPTION CHARACTERS

In order to investigate the absorption characters of the anechoic layers bonded to the steel plate backings, three different anechoic layers are considered in this section. The models of the homogenous slab, the Alberich anechoic layer, and the local resonance slab, which are widely researched for the underwater sound absorption, are analyzed with the FEM. Figures 1(a)–1(c) show the sketches of the three models: Fig. 1(a) represents that of the homogenous slab, Fig. 1(b) denotes the Alberich anechoic layer, and Fig. 1(c) is for the local resonance slab. The incident plane wave enters from the negative direction of \( z \)-axis. During the progress of the calculation, it is assumed that the host extends infinitely on the \( x-y \) plane to avoid the scattering at the border. According the theory mentioned above, a single unit cell is meshed, which follows that a \( \lambda/12 \) criterion is applied on the shear waves and a \( \lambda/4 \) criterion on the flexural wavelength.

A. The homogenous slab model

To describe the absorption mechanism of the anechoic layers with steel plate, the simplest model, a homogenous
slab, is taken into account. The thickness of the slab is 50 mm, and the lattice constant is fixed at 16 mm to reduce the computation with the FEM. Considering the thickness of the submarine hull, the steel slab is 6 mm thick which is selected as Ref. 11. The physical parameters are selected following the article,9 which are assumed not to vary with the frequency. The properties of the rubber material are $E = 1.4 \times 10^8$ Pa, $\rho = 1100$ kg/m$^3$, $\nu = 0.49$, the loss factor for the Young’s modulus is 0.23. The density and sound speed of the fluid are assumed to be 1000 kg/m$^3$ and 1489 m/s. For the steel slab, the Young’s modulus is $2.1 \times 10^{11}$ Pa, the density is 7890 kg/m$^3$, and the Poisson’s ratio is 0.3.

The absorption coefficients of the homogenous slab are calculated by the FEM, the results of which are shown in Fig. 2. The solid curve represents the absorption coefficients versus frequency when the slab is immersed in the water, while the dashed-dotted line denotes the result of the slab bonded to a steel plate with an air termination. It can be found from Fig. 2 that there is a peak around 7.6 kHz on the absorption spectrum of the slab with steel plate. However, for the slab immersed into the water, the absorption coefficients increase with the frequencies, and no absorption peaks occur on the curve in the range of calculated frequencies. One can also readily observe that the homogenous slab immersed in the water has a weak absorption than it is adhered to the steel plate, which is caused by the larger transmittances in the water.

In order to give an intuitive understanding of the absorption mechanism at the frequency of the absorption peak, we calculate the displacement vectors of unit cell on the $zOx$ cross-section plane at this peak frequency shown in Fig. 3. The displacement vectors are denoted by the arrows with length scaled to the displacement amplitude. The directions of the displacement vectors in Fig. 3 indicate that the steel plate vibrates along the direction of the $z$-axis with the equal amplitudes, while the homogenous slab is extended and compressed longitudinally. The homogenous slab and the steel plate is taken as a whole and can be regarded as a mass-spring oscillator in which the mass is offered by the steel plate and the rubber slab plays the role of spring. As the frequency of the absorption peak is influenced by this mass-spring system. The absorption peak can be placed to a lower frequency through varying some structural or physical parameters of the system, such as increasing the thickness of

**FIG. 1.** The sketch of the structure model. The incident plane wave enters from the negative of $z$-axis. (a) The homogenous slab, (b) the Alberich anechoic layer, and (c) the local resonance slab.

**FIG. 2.** (Color online) Comparison of the absorptances of the homogenous slab with different backings. Solid line: water backing. Dash dotted line: steel plate backing.

**FIG. 3.** (Color online) The displacement vectors graph of a unit cell of the homogenous slab on the $zOx$ cross section with the steel plate backings. The incident wave enters the structure from the bottom of the panels. The arrows denote the displacement vectors of the nodes in the model.
the steel plate or decreasing the Young’s modulus of the rubber slab.

B. The Alberich anechoic layer model

The Alberich anechoic layer model, which is a type of rubber coating with air-filled cavities, is widely used to illustrate the absorption mechanism. In this paper, an Alberich anechoic model is selected the same as what is adopted in the literature,\(^9\) where the absorption performance is considered when the coating is adhered to the steel plate. As mentioned above, the thickness of the steel slab is fixed at 6 mm. The Alberich anechoic layer is 20 mm thick with a grating spacing 30 mm, and cylindrical cavities of 15 mm height and 20 mm diameter. The physical parameters for the rubber are the same with those in part A.

Figure 4 compares the absorptances of anechoic coatings with the half-infinite water backing and the steel slab with an air termination. The solid line represents the absorption coefficients when the Alberich anechoic coating is immersed in the water, and the dashed-dotted line denotes the absorptances of the anechoic coating with the steel slab. Similarly to the homogenous slab, one can readily find that the Alberich anechoic coating immersed in the water has a weak absorption than that with the steel plate. From the dashed-dotted line in Fig. 4, it can be found that two peaks, which are round 1680 Hz and 4630 Hz, respectively, occur in the absorption spectrum. For the model immersed in the water, the frequency of the absorptances peak on the solid line locates at about 4600 Hz, which has a good agreement with the frequency of the latter peak appeared in the dashed-dotted line. This phenomenon indicates that the appearances of the absorption peaks at this frequency are for the same reason. It can also be inferred that the absorption peak at the lower frequency is induced by the vibration of the steel plate.

In order to understand the origin of the above absorption peaks, the motions of the Alberich model with the steel plate at the peak frequencies are investigated. The displacement vectors of the model on the \(zOx\) cross-section are shown in Fig. 5. Figure 5(a) indicates the displacement vectors of the model on the cross-section at 1680 Hz, and Fig. 5(b) at 4630 Hz. From Fig. 5(a), one can see that the steel plate and the cover layer at the bottom have the larger amplitudes of displacements than the other parts in the unit cell. The displacement vectors on the steel plate point to the positive \(z\)-axis, and those on the cover layer at the bottom are nearly along the negative direction of \(z\)-axis, which means that the rubber coating vibrates longitudinally in the direction of the incident wave. This vibrating motion of the Alberich anechoic layer with the steel plate is similar to that of the homogenous slab. It also can be found in Fig. 5(a) that the cover layer at the bottom undergoes not only extensional/compressive deformation but also the shear deformation due to the bulge in its central region. It is observed that the rubber of the cavity wall exhibits an in-and-out motion, which is denoted by the acclivitous arrows of the displacement vectors. Therefore, the longitudinal waves undergo a substantial mode conversion into transverse waves through the corresponding shear deformation of the cavity wall during the resonance. The transverse waves dissipate rapidly in viscoelastic medium, enhancing the acoustic energy dissipation. In Fig. 5(b), it is seen that the vibration takes place mainly in the region of the bottom cover layer, while the other parts of the unit cell have a weak oscillation. Compared with the top cover layer bonded to the steel slab, the stiffness of the bottom layer is relative smaller than that of the composite layer. Therefore, the motion of the bottom cover layer will be dominant over the other. Now, it is clearly that the absorption peak at the lower frequency is caused by the resonance of the mass-spring system, which is composed of the steel slab and Alberich anechoic layer, while the peak at the higher frequency is caused by the resonance of the cylindrical cavities in the rubber layer.

![Figure 4](image1.png)

**FIG. 4.** (Color online) Comparison of the absorptances of Alberich anechoic layer with different backings. Solid line: water backing. Dash dotted line: steel plate backing.

![Figure 5](image2.png)

**FIG. 5.** (Color online) The displacement vectors graph of a unit cell of the Alberich anechoic layer on the \(zOx\) cross section with the steel plate backings. The incident wave enters the structure from the bottom of the panels. The arrows denote the displacement vectors of the nodes in the model. (a) The displacement vectors at 1680 Hz, and (b) the frequency of 4600 Hz.
C. The local resonance slab model

Finally, we analyze the absorption mechanism of the local resonance slab with the steel plate. A viscoelastic polymer slab containing a plane of uniform scatterers arranged in a square lattice is considered. Both the lattice constant and the whole slab thickness are 19 mm. The scatterer is a spherical hard core coated by a layer of soft silicon rubber. The core has a 10 mm diameter and the coating thickness is 2.5 mm. In order to reduce the weight of the sample and facilitate the understanding of the effects of the backing, we choose Al material as the core and the thickness of the steel plate as 15 mm. Material parameters for all components are fixed without considering the dependence on frequency for simplicity, and are listed in Table I.

Figure 6 shows the absorption coefficients as a function of frequency, in which the solid line represents the absorptances for the local resonance slab immersed in the water, and the dashed-dotted line denotes those of the local resonance slab bonded to the steel plate. One can find that there is weak absorption peak around 2500 Hz on the solid line. Here the host rubber is water impedance-matched, so the water backing affects little on the absorption of the composite slab. The weak absorption peak is induced by the weak resonance of the scatterers determined by the light Al core, which can be verified by the displacement vectors graph of finite element results (the plots are omitted for paper length). For the steel backing, a distinct feature is that a new absorption peak occurs at about 1000 Hz with a larger amplitude.

In order to gain more physical insight into the variation of the above absorption peaks under the steel backing, it will be useful to illuminate the motion of the unit cell described by the displacement vectors graph at the frequencies of absorption peaks. Figures 7(a)–7(b) represent the displacement vectors on the $zOx$ cross-section at 1070 Hz and 2500 Hz when the slab is bonded to the steel plate. One can obviously find that the hard core vibrates acutely at 2500 Hz in the absorption spectrum, which is denoted by the larger arrows on the nodes of the local resonance unit in Fig. 7(b). It also validates that the first peak on the absorption curves is caused by the resonance of the scatterers evoked by the incident wave. As can be seen from Fig. 7(a), the distribution of the displacement vectors is similar to that of the homogenous slab model at the first absorption peak. The steel plate moves along the direction of z-axis as a rigid slab with the equal amplitudes at every node. The nodes on the steel plate possess the larger displacement amplitudes than the other parts in the unit cell. In other words, the hard core and the soft coating have a relative weak oscillation, while the other parts of the unit cell present a longitudinal motion at 1000 Hz.

IV. DISCUSSIONS AND EXPERIMENTS

A. Discussions

According to above investigations, one may conclude that a new absorption peak will be induced if steel plate backing and the anechoic layer is considered as a whole. The physical mechanism of this phenomenon can be analogous to a mass-spring system, where the anechoic layer and the steel slab are equivalent to the spring and mass, respectively.

![Figure 6](image6.png)

**FIG. 6.** (Color online) Comparison of the absorptances of the local resonance slab with different backings. Solid line: water backing. Dash dotted line: steel plate backing.

Hence, it can be concluded that a new absorption peak is induced by the resonance of the steel plate and the anechoic layer in the local resonance slab model.

TABLE I. Physical parameters of materials for the local resonance slab.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ (kg m$^{-3}$)</th>
<th>Young’s modulus (Pa)</th>
<th>Poisson’s ratio ($\nu$)</th>
<th>Loss factor ($\mu$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>7890</td>
<td>$2.09 \times 10^{11}$</td>
<td>0.275</td>
<td>0</td>
</tr>
<tr>
<td>Al</td>
<td>2690</td>
<td>$7.44 \times 10^{10}$</td>
<td>0.334</td>
<td>0</td>
</tr>
<tr>
<td>Host rubber</td>
<td>1100</td>
<td>$2.97 \times 10^{7}$</td>
<td>0.498</td>
<td>0.3</td>
</tr>
<tr>
<td>Silicon rubber</td>
<td>1300</td>
<td>$2.0 \times 10^{6}$</td>
<td>0.462</td>
<td>0.3</td>
</tr>
</tbody>
</table>

![Figure 7](image7.png)

**FIG. 7.** (Color online) The displacement vectors graph of a unit cell of the local resonance slab on the $zOx$ cross-section with the steel plate backings. The incident wave enters the structure from the bottom of the panels. (a) The displacement vectors on the cross-section at 1070 Hz, and (b) the frequency of 2500 Hz.
Therefore, the frequency of the absorption peak induced by the resonance of the steel plate and anechoic depends on the equivalent stiffness and mass. In order to validate this conclusion, some parameters which can change the equivalent stiffness and mass are discussed.

Keeping the thickness of the steel plate unchanged, a 2-layers local resonance slab which is composed of two layers of uniform local resonance slabs mentioned in Sec. III C is considered. Figure 8 shows the comparison of the absorption coefficients for the different slabs. The solid line in Fig. 8 represents the absorption coefficients for the 2-layers local resonance slab, while the dashed-dotted line indicates those of the 1-layer local resonance slab. Comparing two curves in Fig. 8, one can see that the first absorption peak on the solid line moves to the lower frequency. It is caused by the decreasing of equivalent stiffness offered by the rubber host. When the local resonance slab varies from one layer to two layers, the thickness of the anechoic becomes larger, which will result in a smaller extensional rigidity. It also can be found that the frequency of the absorption peak induced by the local resonance of the scatterers keeps unchanged. The absorbing ability at the frequency of the local resonance is enhanced for the two layers structure, which is indicated by the larger peak amplitude at this frequency for the solid line.

Taking the 2-layers local resonance slab model for example, we change the thickness of the steel plate from 6 mm to 15 mm. Figure 9 shows the comparison of the absorption coefficients under the steel plate backings with different thicknesses. One can readily see that the absorption peak moves to lower frequencies when increasing the thickness of the steel slab. By adding the thickness of the steel plate, the equivalent mass will be increased in the mass-spring system. Therefore, the resonance frequency of the mass-spring system will be reduced, and the absorption peak induced by the resonance moves to lower frequencies, correspondingly.

B. Experiments

According to the physical and structural parameters mentioned in Sec. IV A, the samples of the 2-layers local resonance slab were fabricated. The cross sections of the array of units embedded in the host rubber and the photo of samples are shown as Fig. 10. The radius of the sample is 59 mm to meet well the demands of experimental equipment, and the resonant scatterers within each layer are arranged in a lattice with a mean adjacent distance 19.5 mm. The absorption coefficients are measured by the transfer function method in a standing wave tube.

Figure 11 gives the experimental absorption coefficients of the sample under the steel plate backings with different thicknesses. One can readily see that an absorption peak appears under each backing. The absorption peak moves to lower frequencies when the sample is set in turn under the 10 mm and 15 mm steel backing, respectively. It can be found that the results of calculation under different backing show...
good agreement with those of the experiments. However, the absorption peak induced by the local resonance is smoothed in the experimental results, which may be induced by the variation of the actual parameter with frequency and the bubble scattering, the thorough analysis is out of our present work.

V. CONCLUSIONS

We have investigated the absorption performances of the different anechoic coating models with the steel plate backings by the FEM. The theoretical results in all the models show that a new absorption peak occurs on the absorption spectrum, which is induced by the longitudinal resonance of the anechoic layer and the steel slab. The effects of the steel backing on the acoustic absorption are illuminated by changing the thickness of the steel slab and the anechoic layer. The resonance moves to low frequency range when adding the thicknesses of the steel slab and the anechoic layer, which will result in the same moving trend of the frequency for the absorption peak. And this phenomenon is also validated by the experimental results of the local resonance slab under the steel slabs with different thicknesses.

ACKNOWLEDGMENT

This research is partially funded by the National Natural Science Foundation of China under Grants Nos. 11004249 and 50875255. We are grateful for the excellent advice of the reviewers.