Wave propagation and attenuation in plates with periodic arrays of shunted piezo-patches

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1. Introduction

Elastic wave propagation in periodic structures has been researched for many years [1–3]. The vibration response of periodic structures has been applied primarily to pass band and stop band analysis. Recently, the propagation of elastic or acoustic waves in periodic metamaterials called phononic crystals (PCs) has received considerable attention [4–9]. There are two kinds of gap formation mechanisms for PCs, Bragg scattering mechanism and locally resonant (LR) mechanism. The studies have shown that the existence of the Bragg gaps is strongly connected with a large acoustic impedance ratio between the scatters and the matrix. The center frequencies are always given by Bragg’s condition \( f = n \left( v / 2a \right) \) \( (n = 1, 2, 3, \ldots) \), where \( v \) is the elastic velocity of the matrix material and \( a \) is the lattice constant. The pioneering work of Liu et al. has opened additional fields of PCs [6]. The authors studied three-dimensional PCs consisting of cubic arrays of coated lead spheres immersed in an epoxy matrix, and proposed a new kind of gap formation mechanism, i.e., LR. The resonances of scatter units are the dominant factor in the formation of LR gaps.

With the rapid development of smart materials and structures, more and more intelligent elements are introduced into noise and vibration control systems. Piezoelectric shunt damping technique is one of what receives considerable attentions [10–13]. Structures periodically shunted by electrical resonant circuits can also produce locally resonant band gaps. Moreover, the tunable characteristics of shunted piezo-patches allow the band gaps to be tuned over desired frequency ranges. Thorp et al. used a periodic array of RL-shunted piezos mounted on a rod [14] and fluid-loaded shells [15] to create...
band gaps centered at the tuning frequencies of the shunting circuits. Wang et al. experimentally investigate the low-frequency locally resonant band gaps induced by arrays of resonant shunts with Antoniou's circuit [16]. Airold et al. proposed using multi-resonant shunts to generate multiple locally resonant band gaps in a beam [17]. Subsequently, they proposed to design one-dimensional tunable acoustic metamaterials through periodic arrays of resonant shunted piezos [18]. These studies demonstrate how resonant piezoelectric shunts can be utilized to affect the equivalent mechanical properties of an elastic waveguide and therefore suggest their application for the development of tunable band gaps. Though this strategy has been extended to flat plates hosting periodic arrays of RL-shunted piezo-patches [19,20] for 2 years, little progresses were reported afterwards. The reason for this is that no effective modeling method is proposed so far to predict the dispersion properties in arbitrary direction, though Spadoni et al. [19] have generalized the transfer matrix method to calculate the dispersion relations of waves propagating in some specific directions.

In this paper, we propose an effective method to predict the dispersion relations of waves propagating in any directions. Based on the solution to a transcendental eigenvalue problem, the propagation constants of waves propagating in plates with periodic arrays of shunted piezos are calculated. The influences of electrical resonance and damping to the wave propagation in all directions are study in detail. In Section 2, the approach to band-gap calculation based on finite element modeling is described. A unit cell of the periodic plate is analyzed using conventional finite element method. Then, Bloch theorem is employed to reduce governing equations into transcendental eigenvalue problems, whose solutions yield the propagation constant of waves in any direction. The propagation constants of infinite-cell model are discussed in Section 3. The dispersion relations of the plate with resistive and resistive-inductive shunts are analyzed, respectively. The variations of band gaps with different shunting parameters are discussed. Specifically, wave propagation and attenuation in all directions are investigated at large. The finite element simulation and experimental test of finite-cell model are conducted in Sections 4 and 5, respectively. Finally, some conclusions are given in Section 6.

2. Finite element modeling

2.1. Configuration

The assembled shunting system consists of a substrate plate and arrays of shunted piezoelectric patches, which are oppositely bonded to the upper and lower surfaces of the plate. Each pair of piezo-patches is connected to a shunting circuit with complex impedance referred to as $Z$. The host plate and piezo-patches as a whole behave as a tunable two-dimensional waveguide propagating transverse waves. The wave propagation in the waveguide can be conveniently evaluated through the dynamic analysis of one unit cell (Fig. 1) with the application of Bloch theorem.

2.2. Dynamics of unit cell in discretized format

The electromechanical model of the plate with surface-mounted piezo-patches are based on the stress-charge form of the constitutive equations for a piezoelectric material, which can be expressed as [17]

$$
\begin{bmatrix}
T \\
D \\
S \\
E
\end{bmatrix} =
\begin{bmatrix}
e^c & -e^T \\
e & e^T
\end{bmatrix}
\begin{bmatrix}
T \\
D \\
S \\
E
\end{bmatrix}
$$

(1)

where $T$ is the mechanical stress vector, $D$ the electric displacement vector, $S$ the mechanical strain vector, and $E$ the electric field vector. Also, $e^c$ is the permittivity matrix at constant strain, $e$ the matrix of piezoelectric stress constants, and $e^T$ the stiffness matrix of the piezo-material at constant electric field. In the paper, the superscript 'T' represents the transpose operator.
The combination of host plate and piezo-patches as a laminate is modeled using 4-node Kirchhoff plate elements. Piezoelectric patches are polarized through the thickness direction, with electrodes connected to the top and bottom surfaces. Therefore, the discretized forms of the governing equations of the unit cell are derived by applying Hamilton’s Principle and standard FE procedures, which give [19,20]

\[ M_{uu} \ddot{d} + K_{uu} \dot{d} + K_{\phi u} \dot{\phi} = f \]  

(2)

\[ K_{\phi u} \dot{d} + K_{\phi \phi} \dot{\phi} = q \]  

(3)

where \( \dot{d} \) is the vector of the nodal generalized displacements, \( f \) the vector of the generalized forces, \( q \) the total charges on the electrodes and \( \dot{\phi} \) the electric potential difference between the electrodes of each piezo-patch. Also, \( M_{uu}, K_{uu} \) and \( K_{\phi u} \) are, respectively, the assembled mass, stiffness and coupling matrices. Noteworthy, \( K_{\phi \phi} \) is equal and opposite to the total capacitance of the piezo-pairs at constant strain.

Assuming the harmonic wave motion at frequency \( \omega \), the relationship between \( \dot{\phi}(\omega) \) and \( q(\omega) \) can be given by

\[ q(\omega) = \frac{\dot{\phi}(\omega)}{10Z(\omega)} \]  

(4)

Substituting Eq. (4) into Eq. (3), it can be solved as

\[ \dot{\phi}(\omega) = \left[ \frac{1}{10Z(\omega)} - K_{\phi \phi} \right]^{-1} K_{\phi u} \dot{d} \]  

(5)

Then, substituting Eq. (5) into Eq. (2), the reduced form can be expressed as

\[ D(\omega) \dot{d} = f \]  

(6)

where \( D(\omega) \) is the dynamic stiffness matrix and given by

\[ D(\omega) = M_{uu} - \omega^2 M_{uu} + K_{\phi u} \left[ \frac{1}{10Z(\omega)} - K_{\phi \phi} \right]^{-1} K_{\phi u} \]  

(7)

2.3. Bloch analysis

Any lattice structure in a two-dimensional space can be constructed by translating a repeating unit cell along two independent basis vectors \( \mathbf{e}_1 \) and \( \mathbf{e}_2 \) [21]. If \( R(\mathbf{r}) \) denotes the displacement of a point located a \( \mathbf{r} \) in a reference unit cell, then

\[ R(\mathbf{r}) = R_\mathbf{r} e^{i(\omega t - \mathbf{k} \mathbf{r})} \]  

(8)

where \( R_\mathbf{r} \) is the amplitude and \( \mathbf{k} \) is the wave vector. The integers \( n_1 \) and \( n_2 \) denote the translates of the unit cell along the \( \mathbf{e}_1 \) and \( \mathbf{e}_2 \) directions, such that the point in the cell \( (n_1, n_2) \) corresponding to \( \mathbf{r} \) is located at \( \mathbf{r}_{n_1,n_2} = \mathbf{r} + n_1 \mathbf{e}_1 + n_2 \mathbf{e}_2 \). Letting \( \mu_x = -\mathbf{k} \cdot \mathbf{e}_1 \) and \( \mu_y = -\mathbf{k} \cdot \mathbf{e}_2 \), the displacement at \( \mathbf{r}_{n_1,n_2} \) can be expressed as

\[ R(\mathbf{r}_{n_1,n_2}) = R(\mathbf{r}) e^{i\mu_x n_1 + \mu_y n_2} \]  

(9)

Thus for two cells adjacent along the \( \mathbf{e}_1 \)-axis, \( R(\mathbf{r}_{n_1+1,n_2}) = R(\mathbf{r}_{n_1,n_2}) e^{i\mu_x} \). Similarly, for two cells adjacent along the \( \mathbf{e}_2 \)-axis, \( R(\mathbf{r}_{n_1,n_2+1}) = R(\mathbf{r}_{n_1,n_2}) e^{i\mu_y} \).

According to the location of nodes shown in Fig. 2, the nodal displacements vector \( \mathbf{d} \) can be partitioned into

\[ \mathbf{d} = \begin{bmatrix} d_1 & d_B & d_R & d_{LB} & d_{RB} & d_{LT} & d_{RT} \end{bmatrix}^T \]  

(10)

\[ \mathbf{f} = \begin{bmatrix} 0 & f_B & f_R & f_{LB} & f_{RB} & f_{LT} & f_{RT} \end{bmatrix}^T \]  

(11)

The Bloch theorem in square lattice can be expressed as [21]

\[ \begin{align*}
    d_T &= e^{i\mu_y} d_B, \\
    d_R &= e^{i\mu_x} d_L, \\
    d_{LT} &= e^{i\mu_y} d_{LB}, \\
    d_{RT} &= e^{i\mu_x + \mu_y} d_{LB}
\end{align*} \]  

(12)

Fig. 2. Depiction of nine displacements in a square lattice.
where $\mathbf{u} = [\mu_x \mu_y]$ is the vector of the propagation constants. The propagation constants are complex numbers $\mu_k = \delta_k + i\epsilon_k$ ($k = x, y$), whose real ($\delta_k$) and imaginary ($\epsilon_k$) parts are denote attenuation and phase constants, respectively. They describe the nature of elastic waves propagating in the periodically shunted plate: purely imaginary propagation constants ($\epsilon_k = 0$) correspond to waves free propagating freely, while the existence of a real part indicates that amplitude decay occurs as waves propagate from on cell to the next.

The matrices form of Eqs. (12) and (13) can be given by

$$\mathbf{d} = \mathbf{T}(\mu_x, \mu_y) \mathbf{d}$$

$$\mathbf{T}T(-\mu_x, -\mu_y) \mathbf{f} = 0$$

(14)

where $\mathbf{d} = [d_l \quad d_{lB}]^T$ and

$$\mathbf{T}(\mu_x, \mu_y) = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}^T$$

(15)

Substituting Eq. (14) into Eq. (6) yields

$$[\mathbf{T}T(-\mu_x, -\mu_y) \times \mathbf{D}(\omega) \times \mathbf{T}(\mu_x, \mu_y)] \mathbf{d} = 0$$

(16)

If the direction of wave propagation $\theta$ is specified, the relations of $\mu_\theta$, $\mu_x$ and $\mu_y$ can be expressed as

$$\begin{cases}
\mu_x = \mu_\theta \cos(\theta) \\
\mu_y = \mu_\theta \sin(\theta)
\end{cases}$$

(17)

Substituting Eq. (17) into Eq. (16) gives

$$\mathbf{D}(\mu_\theta, \omega) \mathbf{d} = 0$$

(18)

where

$$\mathbf{D}(\mu_\theta, \omega) = \mathbf{T}^T[-\mu_\theta \cos(\theta), -\mu_\theta \sin(\theta)] \times \mathbf{D}(\omega) \times \mathbf{T}[-\mu_\theta \cos(\theta), -\mu_\theta \sin(\theta)]$$

(19)

Given $\omega$, Eq. (19) becomes a transcendental eigenvalue problem, which cannot be solved by conventional approach. So far, however, a range of numerical methods have been proposed to solve this transcendental eigenvalue problem, such as Newton’s eigenvalue iteration method [22,23], Powell’s method [24], the interval Newton method [25–27] and so on. In the following calculation, a variant of Powell’s method implemented as the function fsolve in Matlab was used [28].

3. Propagation constant of infinite periods

For the plate with infinite cells of shunted piezos, the propagation constant can be utilized to characterize the wave propagation properties. As an example, we choose epoxy (Young’s modulus $E = 4.35 \times 10^6$ N m$^{-2}$, Poisson’s ratio $\nu = 0.37$ and density $\rho = 1.18 \times 10^3$ kg m$^{-3}$) as the material of host plate and PZT-5H as the material of piezo-patches. The geometrical parameters of the unit cell are summarized in Table 1, and the material properties of PZT-5H are listed in Table 2.

Based on the aforementioned finite element method and transcendental eigenvalue solution approach, the attenuation constants and phase constants can be calculated. As the unit cell is square, one only needs to sweep $\theta$ from 0 to $\pi/4$ and gets the propagation constants in arbitrary direction from symmetry (Fig. 3), i.e.

$$\mu_\theta = \begin{cases}
\mu_\theta, & 0 \leq \theta \leq \frac{\pi}{4} \\
\mu_{\pi - \theta}, & \frac{\pi}{4} < \theta \leq \frac{\pi}{2}
\end{cases}$$

(20)

<table>
<thead>
<tr>
<th>$l$ (mm)</th>
<th>$h$ (mm)</th>
<th>$l_p$ (mm)</th>
<th>$h_p$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>5</td>
<td>40</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 1
Geometrical parameters of unit cell.
3.1. Resistive shunts

If each shunting circuit is connected to a single resistor with value $R$ as shown in Fig. 4, the complex impedance $Z$ becomes

$$Z(\omega) = R$$  \hspace{1cm} (21)

Fig. 5 presents the variations of propagation constant with the value of shunting resistance ($\theta=0$). The impedances mismatch between the host plate and piezo-patches generate a Bragg gap in the frequency range of 0–1500 Hz. With the increase of shunting resistance $R$, the upper bounding frequency becomes higher, while the lower bounding frequency does not show a visible change. In order to gain a deeper insight into this phenomenon, the wave modes corresponding to the bounding frequencies are shown in Fig. 6. The wave mode at lower bounding frequency shows that the deformation of piezo-patch is central symmetry, i.e., one half of the patch generates positive charges and the other produces equal negative charges at the same time. As a result, the charges on the electrode counteract each other completely and no net charges remain. As the shunting effect vanishes at the lower bounding frequency, its position will be stationary when increasing the shunting resistance. On the contrary, the wave mode at the upper bounding frequency shows that the deformation in the whole piezo-patch is either extension or compression, so the electric charges generated will have the same sign, totally negative or positive. Consequently, the shunting effect of resistance is quite strong at the upper bounding frequency.

The variations of the band gap with $y$ are illustrated in Fig. 7. Both the upper and lower bounding frequencies increase with $y$, while the lower one increases more rapidly. As a result, the width of the gap decreases with $y$ and even reduces to nil around $\pi/4$. The variations of bounding frequencies lead to directional propagation of waves at certain frequencies. For example, the propagations of waves at 800 Hz and 1000 Hz are obviously blocked in some directions. As shown in Fig. 8, the polar plots present the attenuation constants (800 Hz and 1000 Hz) in all directions.

The influences of shunting resistance to the attenuation constant in different directions are shown in Fig. 9. The damping effect results in attenuations in all directions except some special points. Actually, observing Fig. 7, one can easily find that these special points are corresponding to the lower boundary of the gap where the shunting effect vanishes due to charges counteraction.

3.2. Resistive-inductive shunts

If each shunting circuit is connected to a serial resistive-inductive network as shown in Fig. 10, the complex impedance $Z$ becomes

$$Z(\omega) = R + ioL$$  \hspace{1cm} (22)

where $R$ and $L$ denote shunting resistance and inductance, respectively.

The variations of propagation constant with shunting resistance ($L=30$ mH, $\theta=0$) are shown in Fig. 11. When the plate is shunted by resistive-inductive circuits, the resonances of shunting circuits induce a LR gap besides the Bragg gap. With

<table>
<thead>
<tr>
<th>Material properties of PZT-5 H.</th>
<th>Density (kg m$^{-3}$)</th>
<th>$s_{11}^p$ (m$^2$ N$^{-1}$)</th>
<th>$s_{12}^p$ (m$^2$ N$^{-1}$)</th>
<th>$d_{31}$ (C N$^{-1}$)</th>
<th>$e_{33}^{T}$ (F m$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7500</td>
<td>$16.5 \times 10^{-12}$</td>
<td>$-4.78 \times 10^{-12}$</td>
<td>$-2.74 \times 10^{-10}$</td>
<td>$3.01 \times 10^{-8}$</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 3. Symmetry of the unit cell.

Fig. 4. Resistive circuit.

Table 2
the increase of $R$, the location of LR gap almost keeps stationary for it is only strongly related to the natural frequency of the resonant circuits. However, the magnitude of attenuation constant in the LR gap decreases rapidly when increasing $R$, resulting from that the shunting damp mitigates the resonant effect. On the contrary, the influences of resistive damping to the Bragg gap are negligible.

The variations of propagation constant with shunting inductance ($R = 10\, \Omega$, $y = 0$) are shown in Fig. 12. As the eigenfrequency of shunting circuits decreases when increasing the value of $L$, the location of LR gap comes to lower frequencies, accordingly. However, the maximum attenuation constant in the LR gap increase notably, i.e., the resistive damping effect in the LR gap is weaker at lower frequencies.

The variations of LR gap in different directions are illustrated in Fig. 13. The variations of frequency range with $y$ are minor. Consequently, the LR gap is a complete gap in all directions, which is much different from the aforementioned Bragg gap. However, restricted by the electromechanical coupling coefficient of piezos, the band of LR gap is relatively narrow in low frequency domain, only several tens hertz. Fig. 14 shows the attenuation constant in all directions at 560 Hz, which is located in the LR gap. The variation of attenuation constant in different directions is small, so the directional
attenuation of wave propagation is not as distinctive as that in the Bragg gap. As a result, the waves in LR gap cannot propagate without attenuation in any direction, and they are completely confined to the space around the source.

4. Transmission properties of finite periods

The transmission properties of finite cells are investigated to gain an intuitive representation of the wave propagation properties, instead of the dispersion relation of an infinite system. As an example, the transmission properties of a plate
with 6 x 6 arrays of shunted piezos are calculated by ANSYS. The geometrical and material parameters of each cell are in accordance with those in Section 3. All components of the cells are respectively meshed with the elements listed in Table 3. The final finite element model in ANSYS is shown in Fig. 15.
4.1. Resistive shunts

The variations of transmission properties with the value of shunting resistance are shown in Fig. 16. Comparing Fig. 16(a) with Fig. 5, one can find that the transmission curves show a dip in the Bragg gap (shadowed), i.e., the vibrations whose frequencies are located in the gap are attenuated when transmitted in the direction of \( \theta = 0 \). The increase of shunting resistance marginally extends the width of the transmission dip, while considerable attenuation can be observed outside the gap, which lowers the modal peaks. Fig. 16(b) presents the transmission property in the direction of \( \theta = \pi/4 \). Comparing with Fig. 16(a), no distinct big transmission dip can be observed. Therefore, the vibration whose frequency is located in the Bragg gap can transmit directionally.

### Table 3

<table>
<thead>
<tr>
<th>Component</th>
<th>Element type</th>
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<tbody>
<tr>
<td>Substrate plate</td>
<td>SOLID45</td>
</tr>
<tr>
<td>Piezo-patch</td>
<td>SOLID5</td>
</tr>
<tr>
<td>Inductor</td>
<td>CIRCUIT94</td>
</tr>
<tr>
<td>Resistor</td>
<td>CIRCUIT94</td>
</tr>
</tbody>
</table>

#### Fig. 13.
Variations of the LR gap with \( \theta, L=0.2 \text{ H}, R=10 \text{ } \Omega \) (color spectrum represents attenuation constant).

#### Fig. 14.
Variations of attenuation constant with \( \theta \) at 560 Hz, \( L=0.2 \text{ H}, R=10 \text{ } \Omega \).
4.2. Resistive-inductive shunts

Fig. 17 shows the transmission properties of the plate with resistive-inductive shunts. Compared with the solid curves ($L = 0$), the imaginary curves ($L = 0.15$ H) show a transmission dip in the region of LR gap. As the LR gap is a full gap in all directions, the transmission dip exist in both the directions of $\theta = 0$ and $\theta = \pi/4$. Therefore, the vibration whose frequency is located in the LR gap will be significantly confined in the source and cannot transmit freely in any direction.

5. Experimental investigation

In order to verify the numerical results, experimental tests were conducted on the plate shown in Fig. 18. The plate is made up of $6 \times 6$ arrays of cells and the unit cell configuration replicate the schematic of Fig. 1. The dimensions and materials employed were the same as those listed in Tables 1 and 2.
The plate is excited by a vibration shaker (B&K 4824), which is fed with a white noise with the bandwidth from 0 to 1.6 kHz. The responses are measured by two accelerometers (B&K 4507B). PC&PULSE analysis software and B&K 3560C analyzer are used to analyze the response signals and get the transmission properties.

5.1. Implementation of large inductance

Large inductance is required when a low-frequency LR gap is tuned. However, a significant number of such inductors increase the weight and volume costs of this technique. As a synthetic inductor, Antoniou's circuit (Fig. 19) constitutes a viable solution to this problem [18]. The equivalent inductance of this circuit is given by

\[ L = \frac{R_1 R_3 R_4 C}{R_2} \]  

(23)

with Eq. (23), any inductance values can be obtained by properly selecting the components in the Antoniou's circuit. In practice, we choose fixed values of \( R_1 = R_2 = R_3 = 1000 \Omega \) and \( C = 1 \times 10^{-7} \text{ F} \). Thus, the desired inductance can be obtained by properly tuning \( R_4 \). TLE2141CP opamps capable of operating up to \( \pm 22 \text{ V} \) were used as amplifiers in the Antoniou's circuit.

5.2. Resistive shunts

The experimental results of the plate with resistive shunts are shown in Fig. 20. A relatively wide dip of transmission property emerges in the Bragg gap (\( \theta = 0 \)), and no distinct big dip is existed in the direction of \( \theta = \pi/4 \). The shunting resistance increases the modal damping. The above experimental results agree well with those from finite element simulation in Section 4.1.

5.3. Resistive-inductive shunts

The experimental results of the plate with resistive-inductive shunts are presented in Fig. 21. Comparing the curve of \( L = 0 \) (no resonance) with that of \( L = 0.15 \text{ H} \), one can find that the resonances of resistive-inductive shunts produce...
enormous attenuation in the frequency region of LR gap. As the LR gap is complete gap, the transmission dips emerges in both the propagation directions (\(\theta=0\) and \(\theta=\pi/4\)). The experimental results match well with the results of theoretical (Section 3.2) and finite element simulation (Section 4.2).

6. Conclusions

This paper proposes an effective approach to obtain the arbitrary directional propagation constants of plates with periodic arrays of shunted piezos. The resistive shunts can tune the location and attenuation constants of the Bragg gap. With the increase of shunting resistance, the upper bounding frequency becomes higher, while the lower bounding frequency does not show a visible change. The resistive-inductive shunts can split the dispersion curves and form a LR gap. When increase the shunting resistance, the location of LR gap almost keeps stationary for it is only strongly related to the natural frequency of the resonant circuits, whereas the magnitude of attenuation constant in the LR gap decreases rapidly. As the eigenfrequency of shunting circuits decreases when increasing the inductance, the location of LR gap comes to lower frequencies, accordingly. Moreover, the Bragg gap is directional, whose width varies enormously with directions, while the LR gap almost keeps the same in different directions. Finally, the finite element simulations of multiple-cell model in commercial software and experimental results both validate the theoretical calculations.

Acknowledgments

This work was funded by the National Natural Science Foundation of China (Grant nos. 50905182 and 51175501).

References


