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  What is This?
Suppression of vibration and noise radiation in a flexible floating raft system using periodic structures

Yubao Song, Jihong Wen, Dianlong Yu and Xisen Wen

Abstract
This paper investigates the suppression of vibration and noise radiation in a flexible raft system through the use of periodic structure theory. A floating raft isolation system is constructed. The transfer matrix method, the finite element method and periodic structure theory are combined to calculate the dynamic response of the system and the dispersion relations of the periodic structure. The effects of substructure resonances on vibration and noise radiation are analyzed. Next, the periodic isolation structure is used to suppress the vibration and noise radiation in the floating raft system. The isolation performance of the combination of periodic structures and continuous isolators are investigated. Additionally, the influences of the parameters of the floating raft system on vibration and noise are discussed. The numerical results demonstrate that vibration and noise are significantly suppressed in stop bands and are not worsened in pass bands.

Keywords
Flexible floating raft system, periodic structure, vibration isolation, noise radiation

1. Introduction
The floating raft isolation system is a typical vibration isolation structure that is extensively applied in ships. As a key technique of vibration isolation for ships, particularly in submarines, the floating raft system has drawn much attention in recent years (Huang et al., 2011b; Ma and Zhou, 2008; Nelson, 1982; Niu et al., 2005; Pan and Hansen, 1994; Sun et al., 2010; Xiong et al., 1996). Usually, a floating raft system is composed of machinery, a top isolator, a raft, a bottom isolator and the foundation. In theory, its isolation performance can be as much as 35–40 dB (Ma and Zhou, 2008). However, in practice, the ideal performance cannot always be achieved. This is because in traditional vibration isolation theory some ideal assumptions are employed (Kovacic et al., 2008; Ver and Beranek, 2006), such as neglecting the distributed mass of the isolator, assuming a rigid foundation and modeling machinery as a rigid mass. These assumptions are reasonable at low frequencies. However, when modeling flexible mechanical systems at middle to high frequencies, the use of these assumptions to guide the vibration control process and predict noise radiation can produce noticeable errors.

Actually, the effects of the substructure resonances of flexible mechanical systems caused by isolator inertia and foundation elasticity on vibration and noise have aroused much attention in past decades (Du et al., 2003; Harrison et al., 1952; Lee and Thompson, 2001; Mak and Su, 2003; Sykes, 1960). Du et al. (2003) showed that the internal resonances of viscoelastic isolators increased force transmissibility 20–30 dB, and total sound power radiation may increase 3–22 dB. Zhang et al. (2000) demonstrated that the raft stiffness will affect the isolation performance at middle to high frequencies. Wang et al. (2002) investigated the effects of raft and foundation stiffness on isolation
performance and showed that the effects of the stiffness and damping of the foundation are significant. It can be seen that, when the disturbing frequency is in the middle to high frequency range, the force transmission and noise radiation are significantly increased due to the resonances of the machinery, isolator or foundation.

Some studies have investigated the suppression of these adverse effects. Du et al. (2005) discussed the method of controlling the internal resonances in vibration isolators using passive and hybrid dynamic vibration absorbers. To improve the isolation performance of the traditional floating raft system, Sun et al. (2010) introduced dynamic vibration absorbers into the system, and Hui and Ng (2007) optimized the location of isolators in their research. Overall, existing research mainly focuses on the effect of the substructure resonances on the isolation performance, and has primarily examined the effect of a single substructure. Research on the synthetic effects of multiple structures and the suppression of the harmful effects in the floating raft system needs to be advanced further.

Periodic structures exhibit transmissibility “stop bands”, which are frequency ranges in which the transmissibility of elastic waves is very low. By selecting the parameters properly, it is possible to obtain a periodic structure with desired band gaps. This is of great interest in the field of vibration and noise control. Abundant research has been performed on periodic structures (Bondaryk, 1997; Brillouin, 1946; Mead, 1996; Liu et al., 2000; Ruzzene et al., 2002; Solarni et al., 2003; Wang et al., 2004; Wen et al., 2010; Wen et al., 2009; Yu et al., 2012). Besides, some researchers have investigated the use of periodic structure as a vibration isolator (Asiri et al., 2005). Sackman et al. (1989) demonstrated the potential application of periodically layered structures for high frequency vibration isolation. Szefi et al. (2003a, 2003b) performed a series of studies of periodic structures, including predicting the location of band gaps for isolators, constructing a periodic structure and the continuous isolator are studied. The suppression effects of different combinations of the periodic structure and the continuous isolator are studied. Additionally, the influence of the parameters of the floating raft system on the suppression of vibration and noise radiation is studied.

2. Model and methodology

A floating raft system model is shown in Figure 1. In this model, the isolator is modeled as a one dimensional continuous rod, the raft is modeled as a beam and the machine is modeled as a lumped mass. The flexible foundation is a simply supported plate embedded in an infinite baffle, and one side of the plate is coupled with the fluid. The periodic structure is simplified as a periodic rod and can be used to replace the isolator or combined with the isolation in different forms (such as series or parallel connections). There are two additional assumptions. First, the fluid is assumed to be air so that we can neglect the fluid–structure interaction. Second, the connections between the isolators and the machinery and between the raft and the foundation are assumed to be point connections, and only the force along the center axis is transferred.

2.1. Analytical model of the flexible floating raft system

The transfer matrix model of the system without the periodic structure is given as follows

$$
\begin{bmatrix}
q_{f,1} \\
F_{f,1}
\end{bmatrix}
= T_{I_1,1} T_{I_2,1} T_{n_1} \begin{bmatrix}
q_{m,1} \\
F_{m,1}
\end{bmatrix}
$$

$$
= T_{\text{entire}} \begin{bmatrix}
q_0 \\
F_0
\end{bmatrix},
q_{f,1} = M_{f,1} F_{f,1}
$$

In this equation, $F_{f,1}$ contains the external forces acting on the foundation plate from isolators 3 and 4, and the corresponding displacements of the juctions, respectively; $F_{m,1}$ contains the external forces acting on machines 1 and 2, and the corresponding displacements of the machines, respectively. $T_{I_1,1}, T_{I_2,1}, T_{n_1}$ are the transfer matrices of the machines, bottom isolators, top isolators and middle raft, respectively. $M_{f,1}$ is the displacement mobility.
matrix of the connecting point on the foundation. The expressions for \( \mathbf{T}_m, \mathbf{T}_{1,i,j}, \mathbf{T}_{1,J} \) and \( \mathbf{M}_{IL} \) are given as

\[
\begin{bmatrix}
\mathbf{q}_{IL,R} \\
\mathbf{F}_{IL,R}
\end{bmatrix} =
\begin{bmatrix}
\mathbf{q}_{m,R} \\
\mathbf{F}_{m,R}
\end{bmatrix} =
\begin{bmatrix}
\mathbf{T}_m \\
\mathbf{T}_{IL,1}
\end{bmatrix}
\begin{bmatrix}
\mathbf{q}_{IL,1} \\
\mathbf{F}_{IL,1}
\end{bmatrix}
\]

(2a)

\[
\begin{bmatrix}
\mathbf{q}_{IL,R} \\
\mathbf{F}_{IL,R}
\end{bmatrix} =
\begin{bmatrix}
\mathbf{q}_{IL,1} \\
\mathbf{F}_{IL,1}
\end{bmatrix} =
\begin{bmatrix}
\mathbf{T}_{IL,2} \\
\mathbf{T}_{IL,3}
\end{bmatrix}
\begin{bmatrix}
\mathbf{q}_{IL,2} \\
\mathbf{F}_{IL,2}
\end{bmatrix}
\]

(2b)

\[
\mathbf{T}_m =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
w^2m_1 & 0 & 1 & 0 \\
0 & w^2m_2 & 0 & 1
\end{bmatrix}
\]

(3a)

\[
\mathbf{T}_{1,i} =
\begin{bmatrix}
\ell_{i,11} & \ell_{i,12} \\
\ell_{i,21} & \ell_{i,22}
\end{bmatrix}
\begin{bmatrix}
\cos(k_i l_i) & \sin(k_i l_i) \\
E_iA_i k_i^2 \sin(k_i l_i) & \cos(k_i l_i)
\end{bmatrix}
\]

(3b)

\[
\mathbf{T}_{1,J} =
\begin{bmatrix}
\ell_{1,11} & 0 & \ell_{1,12} & 0 \\
0 & \ell_{1,21} & 0 & \ell_{1,22} \\
\ell_{1,12} & 0 & \ell_{1,22} & 0 \\
0 & \ell_{1,21} & 0 & \ell_{1,22}
\end{bmatrix}
\]

(3c)

\[
\mathbf{T}_{1,b} =
\begin{bmatrix}
\ell_{1,31} & 0 & \ell_{1,32} & 0 \\
0 & \ell_{1,41} & 0 & \ell_{1,42} \\
\ell_{1,32} & 0 & \ell_{1,42} & 0 \\
0 & \ell_{1,41} & 0 & \ell_{1,42}
\end{bmatrix}
\]

(3d)

The transfer matrix \( \mathbf{T}_{1,J} \) of the middle raft is obtained from the finite element (FE) model, in which the raft is considered as an Euler-Bernoulli beam. Thus, we can obtain the mass matrix and the stiffness matrix.
of the raft easily. The discrete dynamic equation of the raft obtained from the FE model is

\[
(K_\tau + i\omega C_\tau - \alpha^2 M_\tau)q_\tau = F_\tau
\]  

(4)

where \(K_\tau, C_\tau, M_\tau\) are the stiffness, damping and mass matrices, respectively, \(F_\tau = [F_{\tau,L} F_{\tau,R}]^T\) is the loading vector and \(q_\tau = [q_{\tau,L}, q_{\tau,R}]^T\) is the vector of the degrees of freedom. \(L, I\) and \(R\) represent the left, interior and right boundaries of the raft, respectively. Assuming that there are no external forces on the interior nodes (i.e. \(F_{\tau,I} = 0\)), and introducing the dynamic stiffness matrix \(D_\tau = K_\tau + i\omega C_\tau - \alpha^2 M_\tau\), equation (4) gives the following matrix equation

\[
\begin{bmatrix}
D_{\tau,LL} & D_{\tau,LR} & q_{\tau,L} \\
D_{\tau,RL} & D_{\tau,RR} & q_{\tau,R} \\
-\overline{D_{\tau,LR}} & -\overline{D_{\tau,RR}} & F_{\tau,L} \\
-\overline{D_{\tau,RL}} & -\overline{D_{\tau,LL}} & F_{\tau,R}
\end{bmatrix} = \begin{bmatrix}
F_{\tau,L} \\
F_{\tau,R}
\end{bmatrix}
\]  

(5)

The interior degrees of freedom can be described by \(q_{\tau,I} = -\overline{D_{\tau,LR}} q_{\tau,L} - \overline{D_{\tau,RL}} q_{\tau,R} + D_{\tau,LL} q_{\tau,L} + D_{\tau,RR} q_{\tau,R}\), and equation (5) can then be written as

\[
\begin{bmatrix}
D_{\tau,LL} & D_{\tau,LR} & q_{\tau,L} \\
D_{\tau,RL} & D_{\tau,RR} & q_{\tau,R} \\
-\overline{D_{\tau,LR}} & -\overline{D_{\tau,RR}} & F_{\tau,L} \\
-\overline{D_{\tau,RL}} & -\overline{D_{\tau,LL}} & F_{\tau,R}
\end{bmatrix} = \begin{bmatrix}
F_{\tau,L} \\
F_{\tau,R}
\end{bmatrix}
\]  

(6)

where

\[
D_{\tau,LL} = \overline{D_{\tau,RR}} = \overline{D_{\tau,LR}} = D_{\tau,LR} = -D_{\tau,RR} = \overline{D_{\tau,LL}}\]  

and

\[
-D_{\tau,RL} = D_{\tau,RL} = -\overline{D_{\tau,LR}} = \overline{D_{\tau,RL}} = -\overline{D_{\tau,LL}} = \overline{D_{\tau,LL}} = D_{\tau,RR} = -D_{\tau,LL}.
\]

Furthermore, according to equation (6) and the displacement continuity and force equilibrium constraints at the junction between the isolator and raft given by \(q_{\tau,R} = q_{F,R\tau}, F_{\tau,R} = -F_{I,R}\), the transfer matrix (TM) model of the middle raft is given as

\[
\begin{bmatrix}
q_{L,R} \\
F_{L,R}
\end{bmatrix} = \begin{bmatrix}
q_{L,R} \\
F_{L,R}
\end{bmatrix} = \begin{bmatrix}
D_{L,LL} & D_{L,LR} & q_{L,L} \\
D_{L,RL} & D_{L,RR} & q_{L,R} \\
-\overline{D_{L,LR}} & -\overline{D_{L,RR}} & F_{L,L} \\
-\overline{D_{L,RL}} & -\overline{D_{L,LL}} & F_{L,R}
\end{bmatrix} = \begin{bmatrix}
D_{L,LL} & D_{L,LR} & q_{L,L} \\
D_{L,RL} & D_{L,RR} & q_{L,R} \\
-\overline{D_{L,LR}} & -\overline{D_{L,RR}} & F_{L,L} \\
-\overline{D_{L,RL}} & -\overline{D_{L,LL}} & F_{L,R}
\end{bmatrix} = \begin{bmatrix}
D_{L,LL} & D_{L,LR} & q_{L,L} \\
D_{L,RL} & D_{L,RR} & q_{L,R} \\
-\overline{D_{L,LR}} & -\overline{D_{L,RR}} & F_{L,L} \\
-\overline{D_{L,RL}} & -\overline{D_{L,LL}} & F_{L,R}
\end{bmatrix}
\]

(7)

Normally, the result of vibration isolation can be described by force transmissibility, insertion loss, vibration level difference and power flow. When the noise radiation caused by the vibration of a structure is considered, the radiated pressure and sound power of the foundation are also employed for evaluating the isolation performance. In this paper, the isolation performance is evaluated through the force transmissibility, power flow, radiated pressure and sound power.

The force transmissibility and power flow can be expressed as

\[
T_F = 20\log_{10}\left(\frac{|F_{T,LL}| + |F_{T,LR}|}{|F_{T,ML}| + |F_{T,MR}|}\right),
\]

(8a)

\[
P_F = \frac{1}{2}\Re(i\omega q_{\tau,L}^* F_{T,LL})
\]

(8b)

where \(\Re()\) represents the real part and \(*\) represents the complex conjugate.

The velocity response for a simply supported plate embedded in an infinite baffle is given as follows

\[
V(x, y, w) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} i\omega \varphi_{mn}(x, y) q_{mn}(w)
\]

(9)

where \(\varphi_{mn}(x, y)\) is the mode shape and \(q_{mn}(w)\) is the modal amplitude of the \((m, n)\) mode, and \(\varphi_{mn}(x, y)\) and \(q_{mn}(w)\) are given as

\[
\varphi_{mn}(x, y) = \frac{2}{\sqrt{\rho_\tau ab}} \sin(m\pi x/a) \sin(n\pi y/b)
\]

and

\[
q_{mn}(w) = \sum_{j=1}^{\infty} \frac{2}{\sqrt{\rho_\tau ab}} \sin(m\pi x/a) \sin(n\pi y/b) \left| w_{mn}(1 + i\eta_j) - w_j \right| F_{T,LL},
\]

respectively.

The radiated pressure is given as (Fahy, 1987; Huang et al., 2011b)

\[
p(r_0, \theta_0, \varphi_0) = \frac{i\rho_0 c_0 k_0}{2\pi} \int_0^a \int_0^b V(x, y) e^{-ikr_0} r dxdy
\]

\[
= \frac{i\rho_0 c_0 k_0}{2\pi} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} i\omega q_{mn}(w) A_{mn}
\]

(10)

where \(p(r_0, \theta_0, \varphi_0)\) is the pressure at a far-field point \((r_0, \theta_0, \varphi_0)\) in spherical coordinates, and

\[
A_{mn} = \frac{2}{\sqrt{\rho_\tau ab}} \frac{e^{-ik_0 r_0}}{r_0} \frac{ab}{m\pi^2}
\]

\[
\times \left[(k_0 \sin \theta_0 \cos \varphi_0)^2 - 1\right] \left[(k_0 \sin \theta_0 \cos \varphi_0)^2 - 1\right] \left[(k_0 \sin \theta_0 \cos \varphi_0 b/m\pi)^2 - 1\right]
\]

The radiated sound power is given as (Du et al., 2003; Huang et al., 2011b)

\[
P(w) = \frac{\rho_0 c_0 k_0}{8\pi^2} \int_0^a \int_0^b \frac{\left| \hat{v}(k_x, k_y, w) \right|^2}{k_x^2 + k_y^2} d k_x d k_y
\]

\[
= \frac{\rho_0 c_0 k_0}{8\pi^2} \int_0^{\pi/2} \int_0^{\pi/2} \left| \hat{v}(k_x, k_y, w) \right|^2 \sin \theta d\theta d\phi
\]

(11)

where \(\rho_0, c_0, k_0 = \rho/c_0\) are the density, speed of sound and acoustic wavenumber of the fluid, respectively.
is the TM of the element of the periodic structure. The TM of the periodic structure is given as
\[
T_{ps,x} = T_{ps,y}^N, T_{ps,e} = \prod_{i=1}^{n} T_{ps,y,i} T_{ps,e,i} = \begin{bmatrix}
\cos(k_i j_l) & -\sin(k_i j_l) \\
E_j A_{j}k_i j_l & \cos(k_i j_l)
\end{bmatrix},
\]
where \(T_{ps,y,i}, T_{ps,e,i}\) are the TM of the \(i\)th periodic element, the TM of elements and the TM of the components of the elements, respectively. \(E_j, \rho_j, \eta_j, A_j, k_j = \frac{w}{\sqrt{E_j/\rho_j}}, l_j\) are the complex Young’s modulus, density, loss factor, area, wavenumber and length, respectively, of the \(i\)th component of the \(j\)th periodic structure. Depending on the position of the periodic structure in the system, the location of the matrix \(T_{ps,j}\) will vary in the TM of the entire system. A transform similar to equations (3(c)) and (3(d)) is carried out. When the periodic structure occurs in parallel with isolators, the resulting TM is given as
\[
T_{ps,j} = \begin{bmatrix}
\frac{1}{\bar{a}} & \frac{1}{b} \\
\frac{1}{\bar{b}} & \frac{1}{\bar{a}}
\end{bmatrix}
\]
where \(\bar{a} = \sum_{i=1}^{n} \alpha_{i,j}, \bar{b} = \sum_{i=1}^{n} \alpha_{i,j}^*, \bar{c} = \sum_{i=1}^{n} \alpha_{i,j}^* \) is the TM of the periodic structure or isolator, and \(n\) is the number of parallel structures.

A band gap model is also developed. When a free wave propagates along the periodic structure, the displacements and forces at the connections satisfy
\[
[q_{L,K+1}^T, F_{L,K+1}^T] = \lambda [q_{L,K}^T, F_{L,K}^T]
\]
where \(\lambda = e^{i \alpha}\) is the propagation constant, \(\mu = a + bi\) is a complex constant, and \([q_{L,K}^T, F_{L,K}^T]^T\) are the nodal displacements and forces in connections \(K\) and \(K + 1\), respectively.

Free wave propagation can be described by the eigenproblem
\[
T_{ps,x} [q_{L,K}, F_{L,K}] = \lambda [q_{L,K}, F_{L,K}]
\]
where \(T_{ps,x}\) is the TM of the element of the periodic structure. \(\lambda_j = e^{\mu_j} = e^{\alpha_+ b h_j}\) are the eigenvalues of the TM, \(n\) is the number of degrees of freedom in the cross section of elements and \(\Phi_j = [q_{L,K}, F_{L,K}]^T\) is the corresponding eigenvector. Additionally, the eigenvalues \(\lambda_j\) come in reciprocal pairs, and correspondingly, \(\mu_j\) occur in positive and negative pairs. Thus, eigenvalues and corresponding eigenvectors can be divided into two sets, \(\lambda_j^+ > 0\) and \(\lambda_j^- < 0\) representing the waves propagated in positive and negative directions, \(\lambda_1 < \lambda_2 < \cdots < \lambda_n, |\lambda_j^+| > 1, |\lambda_j^-| < 1\), representing the waves propagated in positive directions \(\lambda_1 < \lambda_2 < \cdots < \lambda_n, \lambda_1 > 1\) and negative directions \(\lambda_1 < \lambda_2 < \cdots < \lambda_n, \lambda_1 > 1\).

### 3. Numerical results and discussion

The parameters of the floating raft system in Figure 1 are given in Table 1. The far-field pressure is observed at \((-36.2, -25, 86.6)\). The external forces acting on machines 1 and 2 are \(F_{m,L_1} = 1 N\) and \(F_{m,L_2} = 1 N\), respectively.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_1)</td>
<td>100 kg</td>
<td>(\rho_f)</td>
<td>7800 kg/m³</td>
</tr>
<tr>
<td>(m_2)</td>
<td>80 kg</td>
<td>(\mu_f)</td>
<td>0.03</td>
</tr>
<tr>
<td>(k)</td>
<td>0.24 m</td>
<td>(\eta_f)</td>
<td>0.04</td>
</tr>
<tr>
<td>(\eta_h)</td>
<td>0.04</td>
<td>(E_f)</td>
<td>2.1 x 10¹¹</td>
</tr>
<tr>
<td>(E_a)</td>
<td>1.5 x 10⁶</td>
<td>(h)</td>
<td>0.04 m</td>
</tr>
<tr>
<td>(A_1)</td>
<td>0.0515m²</td>
<td>(\sigma)</td>
<td>2 m</td>
</tr>
<tr>
<td>(A_2)</td>
<td>0.0515m²</td>
<td>(b)</td>
<td>1 m</td>
</tr>
<tr>
<td>(A_3)</td>
<td>0.0408m²</td>
<td>(\rho_o)</td>
<td>1.2 kg/m³</td>
</tr>
<tr>
<td>(A_4)</td>
<td>0.0401m²</td>
<td>(c_0)</td>
<td>343 m/s</td>
</tr>
</tbody>
</table>

\((x_1, y_1)\) (0.5, 0.5) \((x_2, y_2)\) (0.5, 1.5)
3.1. Effects of foundation plate, raft and isolator resonances on force transmissibility and radiated sound power

Figure 2 shows the force transmissibility curves of the floating raft system (as shown in Figure 1) in different conditions. In the legend, $f$, $r$ and $I$ identify the foundation, raft and isolator, respectively; $0$ and $1$ represent the ideal case and the practical case, respectively, and $2$ represents an additional set of parameters different from set $1$. For example, "L7: $f-1$, $r-0$, $I-1$" is the transmissibility curve for the system in which the effects of the foundation elasticity and the isolator mass are included but the raft is simplified as a rigid mass.

It can be seen that, the resonances of the isolator, raft and foundation clearly increase the vibration and noise radiation. The practical isolators cause the force transmissibility to be 25–40 dB higher than the ideal isolators at the middle to high frequencies (i.e. above 60 Hz in the model). The effect of the elasticity of the raft is less than the mass of the isolators, but the force transmissibility at resonant frequencies is still increased by as much as 30 dB. The effect of the elasticity of the foundation on the force transmissibility is the smallest.

Table 2. Resonant frequencies of the raft system and its substructures in the 0–400 Hz frequency range.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Mode frequencies (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>System</td>
<td>4.8 12.4</td>
</tr>
<tr>
<td>resonances</td>
<td>h = 0.04 m</td>
</tr>
<tr>
<td>$h = 0.04$ m</td>
<td>123.3 (1,1) 197.3 (2,1) 320.6 (3,1)</td>
</tr>
<tr>
<td>$h = 0.01$ m</td>
<td>30.8 (1,1) 49.3 (2,1) 80.2 (3,1) 104.8 (1,2)</td>
</tr>
<tr>
<td>Foundation</td>
<td>123.3 (2,2) 123.3 (4,1) 154.2 (3,2) 178.8 (5,1)</td>
</tr>
<tr>
<td>resonances and modes</td>
<td>197.3 (4,2) 228.1 (1,3) 246.6 (2,3) 246.6 (6,1) 252.8 (5,2) 277.5 (3,3) 320.6 (4,3) 326.8 (5,3) 376.1 (1,4)</td>
</tr>
<tr>
<td>Raft resonances</td>
<td>266</td>
</tr>
<tr>
<td>Isolator</td>
<td>77 141.5 212.5 283 354</td>
</tr>
</tbody>
</table>

Figure 2. Force transmissibility curves of the floating raft system for different cases. For observing more clearly, the plots are assigned to two figures, that is, (a) and (b).
frequencies. The effects are most obvious at middle to high frequencies but are also present at low frequencies.

3.2. Suppression of vibration and noise radiation by the use of the periodic structure

The parameters of the periodic structures from sections 3.2, 3.3 and 3.4 are listed in Table 3. Figure 4 shows the band structure and vibration transmissibility of the periodic structures. In the figure, the shaded area is the band gap, and $\lambda$ is the propagation constant; the dotted lines represent conduction with damping, and the solid lines represent conduction without damping; and the bold lines represent another type of conduction in periodic structure 1 (PS1) in which the components have a different order. Order 1 and order 2 denote different orders of PS1, and $w_d$ and $w_{od}$ represent the structures with and without damping, respectively. It can be observed that, in the 0–400 Hz frequency range, the band gaps of PS1 are 79–90 Hz, 125–300 Hz and 310–400 Hz. The band gaps of periodic structure 2 (PS2) are 75–200 Hz and 225–395 Hz, which is not shown here. In this model, the effect of damping on attenuation is concentrated in pass bands but limited in stop bands. When the viscous damping is included, the phase change in a single period is neither 0 nor $\pi$.

Figure 5 shows the force transmissibility, the power flow transferred to the foundation, the radiated pressure and the radiated sound power of the foundation for different positions of the periodic structure. In the legend, PS and CR represent the periodic structure and the isolator simplified as a continuous rod, respectively, and 1 and 2 represent two sets of parameters. The characters before and after the dash (—) represent the vibration isolation structure located between the machinery and the raft and between the raft and the foundation, respectively.

It can be observed that, in the stop bands, the vibration and noise are significantly reduced. Additionally, even in the pass bands at the frequencies behind the first stop band, vibration and noise radiation are sometimes

![Figure 3](image-url)  
**Figure 3.** (a) Power flow transferred to the foundation and (b) radiated sound power of the foundation.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Density (kg/m$^3$)</th>
<th>Young's modulus (N/m$^2$)</th>
<th>Loss factor</th>
<th>Area (m$^2$)</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS1 (four components)</td>
<td>$\rho_{11} = 1300$</td>
<td>$E_{11} = 1.5 \times 10^6$</td>
<td>$\eta_{11} = 0.08$</td>
<td>$A_{11} = A_{13}$</td>
<td>$h_{1} = 0.06$</td>
</tr>
<tr>
<td></td>
<td>$\rho_{12} = 7800$</td>
<td>$E_{12} = 2.1 \times 10^{11}$</td>
<td>$\eta_{12} = 0.04$</td>
<td>$A_{12} = A_{14}$</td>
<td>$h_{2} = 0.01$</td>
</tr>
<tr>
<td></td>
<td>$\rho_{13} = 1300$</td>
<td>$E_{13} = 1.5 \times 10^6$</td>
<td>$\eta_{13} = 0.08$</td>
<td>$A_{12} = 2A_{11}$</td>
<td>$h_{3} = 0.045$</td>
</tr>
<tr>
<td></td>
<td>$\rho_{14} = 7800$</td>
<td>$E_{14} = 2.1 \times 10^{11}$</td>
<td>$\eta_{14} = 0.04$</td>
<td>$h_{4} = 0.01$</td>
<td></td>
</tr>
<tr>
<td>PS2 (two components)</td>
<td>$\rho_{21} = 1300$</td>
<td>$E_{21} = 1.5 \times 10^6$</td>
<td>$\eta_{21} = 0.08$</td>
<td>$A_{22} = 2A_{21}$</td>
<td>$h_{21} = 0.07$</td>
</tr>
<tr>
<td></td>
<td>$\rho_{22} = 7800$</td>
<td>$E_{22} = 2.1 \times 10^{11}$</td>
<td>$\eta_{22} = 0.04$</td>
<td>$h_{22} = 0.015$</td>
<td></td>
</tr>
</tbody>
</table>

Note: Areas are calculated from the total stiffness of the PS, which is equal to the stiffness of the isolator in the corresponding position.
lower than when periodic structures are not used. In the pass band below the first band gap, although the modes are increased by adopting the periodic structure, the suppression is not reduced. When periodic structures are used in both the top and bottom stages, on one hand, there is greater vibration and noise reduction at the active frequencies of the periodic structure. On the other hand, the amplitude of the main peaks at low frequencies is increased. We also find that, the effect of the periodic structure is significantly affected by its position. If the position is improper, the effect of the periodic structure may be weakened and may even be reduced.

Figure 4. (a) Band structure and (b) vibration transmissibility of PS1.

Figure 5. Effect of the inclusion of the periodic structure on (a) force transmissibility, (b) power flow, (c) radiated sound pressure and (d) radiated sound power.
worse than that of the homogeneous structure. Besides, when the resonant properties of the top- and bottom stage isolators are different, although the number of modes is increased, the amplitude of the peaks is reduced.

Overall, the increasing of vibration and noise caused by foundation plate, raft and isolator resonances is greatly suppressed by the use of the periodic structure, especially in the stop bands. Relatively, in the research of Huang et al. (2011b), only the suppression of the adverse effect of foundation resonances is investigated. Besides, the isolator is modeled as an ideal spring, so the effect of the standing waves of the isolator is not considered.

3.3. Suppression of vibration and noise radiation for various combinations of periodic structures and homogenous isolators

Figure 6 shows the vibration and noise radiation properties of the system for various combinations of periodic structures and combinations of periodic structures and homogenous isolators. In the legend, sc and pc represent series connection and parallel connection, respectively. Otherwise, the notation is the same as Figure 5. As an example, L4 is the case in which the parallel structure is used in the top stage and a pure isolator is used in the bottom stage.

It can be seen that, vibration and noise is reduced effectively in the stop band for each combination. When PS1 and PS2 are used as the top- and bottom stage isolators, respectively, in the system, the active frequency is widened effectively. That is, through careful design, the performance can be improved by inserting different periodic structures in the vibration transmission path. The curves also demonstrate that, when a periodic structure is combined in series with a homogenous isolator, the suppression effect of the periodic structure is weakened slightly. The main reason for this weakening is the presence of the modes of the periodic structure and the top stage isolator together in the transmission path. On one hand, when a periodic structure is used in parallel with a homogenous isolator, the effect is weaker in the stop bands than when a pure periodic structure or a periodic structure and isolator in series are used. On the other hand, a better result is obtained in the pass band below the first stop band.

3.4. Effects of structural parameters on the suppression of noise radiation

Figure 7 shows the effects of the structural parameters of the isolator on sound power. It can be seen that, increasing the Young’s modulus of the isolator shifts the resonant frequencies to higher frequencies, and the disadvantageous effect of isolator resonances is decreased at low frequencies. Besides, when the Young’s modulus of the periodic structure is increased, the middle frequencies of the stop bands are shifted to higher frequencies and the bands are widened. The effect of decreasing the density is similar to that of increasing the stiffness. It can also be found that, although the response of the system is changed by the variety of the parameters of the isolator, the suppression effect of the periodic structure is maintained.

Figure 8 shows the suppression effect of the periodic structure for different structural parameters. The effects of the thickness of the foundation plate and the Young’s modulus of the raft are investigated. It can be observed from Figure 8(a) that the foundation flexibility greatly affects the noise radiation level. Whether a homogenous isolator or a periodic structure is used,
decreasing the thickness of the foundation plate is disadvantageous for the control of noise radiation, especially at low frequencies. Figure 8(b) shows that the Young’s modulus of the raft affects the dynamic properties primarily at middle to high frequencies, and the effect is most obvious near resonant frequencies. In this model, decreasing the Young’s modulus of the raft is advantageous outside the zones near the resonant frequencies. It should be noted that, this conclusion may be affected by the installation condition, the frequency ranges of interest and the parameters of other structures.

Overall, the effect of the parameters when a periodic structure is used is similar to the effect without a periodic structure. Although the parameters affect the dynamic properties of the floating raft system, it can still be concluded that using the periodic structure in flexible mechanical systems helps to suppress vibration and noise radiation. Additionally, the observations are only made from the sound power curves, but they are not changed when the force transmissibility, the power flow and the sound pressure are used to evaluate the isolation performance.

4. Conclusions
In this paper, the use of a periodic structure is presented to suppress the disadvantageous effects of substructure resonances on vibration and noise radiation in a flexible floating raft system. The transfer matrix method (TMM) and finite element method (FEM) are used to construct a model of the flexible floating raft system, and TMM and periodic structure theory (PST) are combined to calculate the dispersion relations and vibration response of the periodic structure. The vibration and noise radiation of the ideal and the flexible floating raft system are analyzed and compared. The results show that substructure resonances significantly
increase the vibration and noise radiation of the system. Thus, the periodic structure is introduced to suppress the disadvantageous effect of the substructure resonances. The isolation performance of various combinations of periodic structures and continuous isolators is investigated.

Numerical results indicate that the increase in vibration and noise caused by the substructure resonances is reduced by using the periodic structure, especially in the stop bands. The component and location of the periodic structure have a significant effect on the results of using the periodic structure. Additionally, the effects of the parameters of the periodic structure and the substructures of the floating raft system on the isolation performance are investigated. Although the structural parameters greatly affect the dynamic properties of the floating raft system, it can still be concluded that the vibration and noise radiation are effectively reduced by introducing the periodic structure to the flexible floating raft system.

Funding
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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_m$</td>
<td>Transfer matrix of the machines</td>
</tr>
<tr>
<td>$T_{1,b}$</td>
<td>Transfer matrix of the bottom isolators</td>
</tr>
<tr>
<td>$T_{1,t}$</td>
<td>Transfer matrix of the top isolators</td>
</tr>
<tr>
<td>$T_{1,r}$</td>
<td>Transfer matrix of the middle raft</td>
</tr>
<tr>
<td>$M_{1,l}$</td>
<td>Displacement mobility matrix</td>
</tr>
<tr>
<td>$F_{m,1,1}, F_{m,1,2}$</td>
<td>External forces acting on machines 1 and 2</td>
</tr>
<tr>
<td>$i$</td>
<td>Imaginary unit</td>
</tr>
<tr>
<td>$w$</td>
<td>Angular frequency</td>
</tr>
<tr>
<td>$m_1, m_2$</td>
<td>Masses of machines 1 and 2</td>
</tr>
<tr>
<td>$l_k, E_k, \rho_k, \eta_k, A_k$</td>
<td>Length, complex Young’s modulus, density, loss factor and cross-sectional area</td>
</tr>
<tr>
<td>$\rho, E, \mu, \eta$</td>
<td>Density, complex Young’s modulus, Poisson’s ratio and loss factor of the foundation</td>
</tr>
<tr>
<td>$h, a, b$</td>
<td>Thickness, length and width of the plate</td>
</tr>
<tr>
<td>$\varphi_{mn,i}, \omega_{mn}$</td>
<td>Mode shape and natural frequency of the $(m, n)$ mode</td>
</tr>
<tr>
<td>$(x_j, y_j), j = 1, 2$</td>
<td>Positions of the connection points</td>
</tr>
<tr>
<td>$K_r, C_r, M_r$</td>
<td>Stiffness, damping and mass matrices of the middle raft</td>
</tr>
<tr>
<td>$F_r$</td>
<td>Loading vector</td>
</tr>
<tr>
<td>$q_r$</td>
<td>Vector of the degrees of freedom</td>
</tr>
<tr>
<td>$D_r$</td>
<td>Dynamic stiffness matrix</td>
</tr>
<tr>
<td>$T_F$</td>
<td>Force transmissibility</td>
</tr>
<tr>
<td>$P_F$</td>
<td>Power flow</td>
</tr>
<tr>
<td>$V(x, y, w)$</td>
<td>Velocity response</td>
</tr>
<tr>
<td>$q_{mn}(w)$</td>
<td>Modal amplitude of the $(m, n)$ mode</td>
</tr>
<tr>
<td>$p(r_0, \theta_0, \varphi_0)$</td>
<td>Sound pressure at a far-field point</td>
</tr>
<tr>
<td>$p(w)$</td>
<td>Radiated sound power</td>
</tr>
<tr>
<td>$\rho_0, c_0, k_0$</td>
<td>Density, speed of sound and acoustic wavenumber of the fluid</td>
</tr>
<tr>
<td>$i(k_r, k_f, w)$</td>
<td>Velocity wavenumber transform</td>
</tr>
<tr>
<td>$E_{ji}, \rho_{ji}, \eta_{ji}, A_{ji}, k_{ji}, l_j$</td>
<td>Complex Young’s modulus, density, loss factor, area, wave-number and length of the $i$th component of the $j$th periodic structure</td>
</tr>
<tr>
<td>$T_{p_{n,e}}$</td>
<td>Transfer matrix of the element of the periodic structure</td>
</tr>
<tr>
<td>$T_{p_{n,i}}$</td>
<td>Transfer matrix of the $j$th periodic structure</td>
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<tr>
<td>$N_f$</td>
<td>Number of the periodic element</td>
</tr>
<tr>
<td>$T_{p_{n,e}}$</td>
<td>Transfer matrix of elements</td>
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<tr>
<td>$\lambda$</td>
<td>Propagation constant</td>
</tr>
<tr>
<td>$\lambda_+^+, \chi_+^+$, $\lambda_-^-, \chi_-^-$</td>
<td>Eigenvalues and corresponding eigenvectors</td>
</tr>
</tbody>
</table>

References


