On the coupling of resonance and Bragg scattering effects in three-dimensional locally resonant sonic materials

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Abstract

Three-dimensional (3D) locally resonant sonic materials (LRSMs) are studied theoretically for purpose of optimising their sub-wavelength performance by coupling resonance and Bragg scattering effects together. Through the study of effective sound speeds of LRSMs, we find that the starting frequency of Bragg scattering can be shifted to sub-wavelength region by softening coats of resonators when the matrix is a low shear-velocity medium. A similar result can be achieved by compressing the lattice constant. By using a layer-multiple-scattering method, we investigate the complex band structure and the transmission spectrum of an LRSM whose Bragg gap is already close to the resonance gap in frequency. The wave fields of the composite simulated by COMSOL are further analysed at several typical frequencies. The result shows that the approaching of two kinds of gaps not only broadens the bandwidth of the resonance gap, but also increases the depth of the Bragg gap since the interaction between resonant modes and scattering waves are enhanced. By varying the shear velocity of coats, we obtain a coupled gap, which exhibits a broad transmission gap in the sub-wavelength region. When the loss of coats is considered, the coupled gap can not only maintain a good sound blocking performance, but also perform an efficient absorption in the low frequency region.

1. Introduction

The locally resonant sonic material (LRSM) is a structural composite with local resonators (LRs) periodically embedded in an elastic matrix, which is initially proposed by Liu et al. [1]. The LRs are typically dense metallic spheres coated with soft materials. Such a composite has prominent sound reflecting or absorbing (if loss included) behaviours around the natural frequencies of LRs, while the wavelength of sound in the matrix is still much larger than the periodicity of the configuration. Owing to the sub-wave-length acoustic behaviour, LRSMs have become the subject of considerable researches since their introduction [2–14]. Especially for the underwater acoustic materials, three-dimensional (3D) LRSMs are ideal candidates for controlling the low-frequency underwater sound [15–21]. Most of these studies have been performed on the existence of resonance band gaps [4–8,15], as well as the corresponding sound transmission [9–12,16] or absorption [17–21] properties. Since the resonance band gap of an LRSM is usually confined to the low-frequency region, few researches have discussed coherent interactions among different LRs provided by Bragg scattering, which usually arises in a higher frequency range.

Consequently, the periodic arrangement of the LRSM has not been fully utilized.

In addition, since local resonators can only have strong reactions when the frequencies of waves are near their natural frequencies, the narrowness of the bandwidth of resonance gap remains a common shortage for LRSMs. Consequently, a variety of studies have been dedicated to broadening the bandwidth of resonance gaps with different strategies [22–27]. For example, Ho et al. tried to enlarge the transmission gap of a locally resonant sonic shield by combining several layers of LRs with different natural frequencies [22]. Meng et al. extended such method and optimised the sound absorbing performance of LRSMs by utilizing the genetic algorithm to adjust the composite's parameters [23]. Larabi et al. proposed a new type of resonators with multi-coaxial cylindrical inclusions in order to enrich the resonant modes of a phononic crystal [24]. A similar research was also found for airborne sound [25]. Gu et al. optimised the band gap of LRSMs by tuning the shape of resonators [26]. All these findings give us a way to broaden the bandwidths of resonance gaps to a certain extent. However, the periodic arrangement of LRSMs has still not been exploited. The combination of resonant frequencies or modes is just a simple accumulation of different resonators in LRSMs. Resonators are found to vibrate independently. Another method for widening the transmission gap of phononic crystals is proposed...
by Sainidou et al. [27] This work involved (stacking or size) disorders in the periodic phononic crystals, which generated Anderson localization in the pass band region, and thereby extended the bandwidth of transmission gaps. However, this method requires sufficient periodicity of unit layers (dozens to hundreds), which is too thick in the application of underwater acoustic materials.

Recently, a theoretical investigation on the coupling of resonance and Bragg band gaps in a one-dimensional (1D) locally resonant elastic system (LRES) [28] was described by Xiao et al. This system consists of a taunt uniform string with periodically attached spring-mass resonators. The analysis demonstrated that both types of band gaps can be enhanced when they are close to each other in frequency or coupled together. More studies on the coupling of different band gaps in locally resonant beams, rods and plates were carried out sequentially [29–31]. Advantages were found for broadening the bandwidth of band gaps by coupling them together, but most of these researches were based on ideal discrete 1D or 2D systems, which are unpractical for the underwater acoustic materials. Until recently, few studies have been found on the investigation of coupling resonance and Bragg scattering effects in 3D LRSMs.

Therefore, this paper devotes its effort to the coupling of different effects of 3D LRSMs, for the purpose of optimising their sub-wavelength acoustic performance. In Section 2, we briefly describe the configuration of the LRSM, the numerical method and the simulation technique. In order to shift Bragg scattering effect to the sub-wavelength region, we first analyse how to reduce the effective sound speed of an LRSM in Section 3. The interaction between the resonance and Bragg scattering effects of 3D LRSMs is discussed explicitly in Section 4. Finally, Section 5 discusses the sound blocking and absorbing performance of LRSMs when coats of LRs include damping.

2. Modelling

This paper focuses on 3D LRSMs, which have periodicities in all three principal directions. Fig. 1a shows a section view of an LRSM with the cutting plane normal to the x-direction and crossing through the centres of the local resonators (LRs). Such a composite consists of finite layers of LRs in the z-direction but infinite rows and columns of LRs in the x- and y-directions. The composite is immersed in water with plane waves normally incident on its left boundary. Resonators in each layer are arranged periodically on a square lattice, whose unit vectors lie parallel to the x- and y-directions, and the normal vector of each layer is parallel to the z-direction. Only cube and rectangular cells are considered in this paper, so there are no offsets between the different layers of LRs. Conveniently, the periodicity in the z-direction is called the layer thickness (l); the periodicities in x- and y-directions are called the lattice constant (A); the radius of the core is denoted by R; and the thickness of the coat is denoted by H (see Fig. 1a). As an initial case, the matrix is a kind of natural rubber (NR1) in which LRs are periodically embedded with L = 20 mm and A = 15 mm. The resonators are steel spheres (R = 4 mm) coated by silicone rubber (SR1) with 1 mm thick. Such an LRSM is named Case I. Table 1 shows the acoustic properties of materials used in this paper. To begin with, we neglect the damping in all the components of the composite. The choices of materials and the structural configurations of LRSMs for different cases are listed in Table 2.

By using a layer-multiple-scattering (LMS) method [32], we can calculate complex band structure and transmission spectrum of the LRSM. The complex band structure demonstrates relations between the Bloch wave vector and the frequency of propagating waves in a phononic crystal with infinite layers of LRs. Correspondingly, we use the term ‘infinite crystal’ to indicate the infinite counterpart of the composite. It is worth noting that the complex band structure considered in this paper only describes the directional dispersion relations when \( k_y = 0 \), where \( k_y \) is the component of the Bloch wave vector reduced in the surface Brillouin zone of the 2D periodic lattice in the xy-plane. In contrast, the transmission spectrum indicates the energy ratio of the transmitted wave to the incident wave for an LRSM with finite layers of LRs. Here and throughout the paper, when we refer to a finite composite, we mean an LRSM composed of 4 layers of LRs in z-direction. Such a periodicity can provide a considerable Bragg scattering while prevent the composite from being too thick for a practical application. During the calculation of the LMS method, the cut-off value of the angular momentum for the spherical-wave expansions is taken to be 4, and the number of considered reciprocal-lattice vectors is set to be 45; this ensures the errors of results less than 1%. Furthermore, we investigate the wave fields of such composites in different frequencies by using a finite element method (FEM) with COMSOL. The simulated model also consists of 4 layers of LRs in z-direction and infinite periods of LRs in x- and y-directions by using the periodic boundary condition. Free tetrahedral elements are used to mesh the composite and the surrounding media, which is shown in Fig. 1b, with the Nyquist frequency up to 50 kHz by limiting the maximum sizes of elements in different materials.

Fig. 2a and b show the real and imaginary parts of the z-component of the Bloch wave vector which has been reduced to the first Brillouin zone (reduced by \( l/\pi \)), and Fig. 2c shows the log-scaled plot of the transmission spectrum. They share the same y-axis which represents the non-dimensional frequency \( (\Omega = 2\pi f/c_{\text{mat}}) \), where \( c_{\text{mat}} \) is the longitudinal velocity of the matrix (1540 m s\(^{-1}\)), and \( L \) is the layer thickness (20 mm). When \( \Omega = 1 \), one half-wave-length of longitudinal waves in the matrix is equal to the layer thickness, which occurs at a frequency of 38.5 kHz for the cases considered in this paper. On account of the symmetry of the composite in z-direction, the negative part of the band structure is not shown as it is symmetric with respect to the positive part.

As shown in Fig. 2a and b, there are two band gaps over the displayed frequency range, which are shaded in light grey. According to the different characteristics of resonance and Bragg band gaps [7], the first band gap in the frequency region of \([0.18,0.21]\) is produced by the local resonances, which is also known as the hybridization band gap [8]. This can be proved by the fact that the reduced Bloch wave vector \( (k_y L) \) has a zero jump for its real part inside the band gap and exhibits a cusp (the slope changes abruptly) for its imaginary part at the corresponding frequency. Actually, the pinnacle of the imaginary part depends on the natural frequency of LRs, at which resonators vibrate most fiercely. In contrast, the second band gap in the frequency region of \([0.94,1.00]\) is the consequence of Bragg scattering by both wave modes. This can be traced by the fact that the real part of the Bloch wave vector is pinned to zero inside the band gap. Meanwhile, its imaginary part varies continuously and smoothly throughout the whole band gap with a maximum value occurring near the mid-frequency of the band gap \( (\Omega = 0.97) \).

Owing to the existence of shear dispersion curves, band gaps can only exist when neither wave modes can propagate in the same frequency range. However, if such an LRSM is located in a fluid medium with the plane waves normally incident on its surface, the pass bands of shear waves will not be stimulated [9]. This can be verified by comparing the imaginary part of the band structure with the transmission spectrum of Case I. As shown in Fig. 2c, the composite has a transmission gap which varies continuously in the frequency region of \([0.75,1.04]\), even though two shear dispersion curves exist in the corresponding band structure. Evidently, this transmission gap coincides with the longitudinal forbidden band induced by Bragg scattering. To avoid ambiguity, we adopt the term ‘Bragg gap’ to indicate the transmission gap induced by
Bragg scattering, while ‘Bragg band gap’ to indicate the Bragg scattering band gap. The same convention is also applied to the transmission gap and band gap induced by resonance. In addition, the composite exhibits a sharp transmission gap in the frequency region of [0.17, 0.22], whose profile is similar to $\text{Im}[K_L]$ in the resonance band gap. This is because both $\text{Im}[K_L]$ and $\log_{10} T$ describe the attenuating efficiency of the propagating waves logarithmically. The difference is the former represents the efficiency for one layer, whereas the latter for four layers of LRs. Moreover, Fig. 2c also shows the transmission spectrum calculated by FEM, which agrees perfectly with the spectrum calculated by LMS method. Hence, we can further investigate the wave fields of the composite at certain typical frequencies according to the FEM’s result.

Fig. 3 shows the wave fields in the $yz$-plane of Case I calculated by COMSOL. In the simulation, a plane wave encounters the left boundary of the matrix and radiates to the surroundings on both sides. The wave fields are painted with hot-scale, with red meaning maximum displacement and white meaning no movement. Arrows indicate the direction of the displacement at each point. Their lengths are logarithmic with the amplitude of the displacement for a better view of the direction when the amplitude is small. As shown in Fig. 3a, when the frequency lies in the resonance gap, i.e. $\Omega = 0.18$, the incoming wave stimulates the first layer of LRs to vibrate fiercely with the phase being opposite to the matrix. Meanwhile, most part of the incoming wave is obstructed at the left side of the first layer of LRs. Fig. 3b shows the wave field when the frequency lies in the Bragg gap ($\Omega = 0.95$). Obvious distinctions between the two transmission gaps are found by comparing their wave fields. As for the Bragg gap, the incoming wave attenuates gradually when passing through different layers of LRs, whereas all the cores of LRs are motionless. This implies that the resonators are not stimulated by the multiple scattering waves in the matrix.

<table>
<thead>
<tr>
<th>Case No.</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>The material of the core</td>
<td>Steel</td>
<td>Steel</td>
<td>Steel</td>
</tr>
<tr>
<td>The material of the coat</td>
<td>SR1</td>
<td>SR1</td>
<td>SR2</td>
</tr>
<tr>
<td>The material of the matrix</td>
<td>NR1</td>
<td>NR2</td>
<td>NR2</td>
</tr>
<tr>
<td>The material of the ambience</td>
<td>Water</td>
<td>Water</td>
<td>Water</td>
</tr>
<tr>
<td>Radius of the core ($R$) (mm)</td>
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<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Thickness of the coat ($H$) (mm)</td>
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<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Lattice constant ($A$) (mm)</td>
<td>15</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Layer thickness ($L$) (mm)</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 2
The chosen materials and the structural configurations of different cases.

Table 1
The acoustic properties of the materials used in this paper.

<table>
<thead>
<tr>
<th>Type of material</th>
<th>Steel</th>
<th>SR1</th>
<th>SR2</th>
<th>NR1</th>
<th>NR2</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass density (kg m$^{-3}$)</td>
<td>7840</td>
<td>1039</td>
<td>1039</td>
<td>1300</td>
<td>1300</td>
<td>1000</td>
</tr>
<tr>
<td>Speed of longitudinal wave (m s$^{-1}$)</td>
<td>5923</td>
<td>185</td>
<td>185</td>
<td>1540</td>
<td>1540</td>
<td>1480</td>
</tr>
<tr>
<td>Speed of shear wave (m s$^{-1}$)</td>
<td>3300</td>
<td>83</td>
<td>122</td>
<td>800</td>
<td>200</td>
<td>Nil</td>
</tr>
</tbody>
</table>

* SR represents silicone rubber.  
* NR represents natural rubber.

Bragg scattering, while ‘Bragg band gap’ to indicate the Bragg scattering band gap. The same convention is also applied to the transmission gap and band gap induced by resonance.

In addition, the composite exhibits a sharp transmission gap in the frequency region of [0.17, 0.22], whose profile is similar to $\text{Im}[K_L]$ in the resonance band gap. This is because both $\text{Im}[K_L]$ and $\log_{10} T$ describe the attenuating efficiency of the propagating waves logarithmically. The difference is the former represents the efficiency for one layer, whereas the latter for four layers of LRs. Moreover, Fig. 2c also shows the transmission spectrum calculated by FEM, which agrees perfectly with the spectrum calculated by LMS method. Hence, we can further investigate the wave fields of the composite at certain typical frequencies according to the FEM’s result.
Hence, the initial case neither fully utilizes the periodic arrangement of LRSMs in the resonance gap, nor the resonant modes of LRs in the Bragg gap. In order to broaden the bandwidth of both transmission gaps, we try to couple them together.

3. Investigation of the effective sound speed of LRSMs

Before discussing the coupling of transmission gaps, we have to know how to pull down the frequency region of the Bragg scattering effect. As is known to all, the necessary condition for the first-order Bragg scattering is that the periodicity of structure equals the half-wavelength of scattering waves ($\lambda = 2L$). Therefore, the starting frequency of the first-order Bragg scattering relates to the layer thickness ($L$) and the sound speed of the composite ($c$) in the form of $f_{BR} = c/2L$. The easiest way to reduce the frequency of Bragg scattering is to increase the layer thickness. However, it is not practical to use a bulky LRSM as an underwater acoustic material. An alternative way is to decrease the effective sound speed of the composite. Through the research, we find that if LRs adopt soft materials as their coats, they can significantly reduce effective sound speed for certain LRSMs. Hence, we evaluate the static effective sound speed of the composite ($c_E$) by using an effective-medium theory [33]. In the long-wavelength limit, the static effective mass density can be obtained by the following formula:

$$\rho_E = \varphi_M \rho_M + \varphi_C \rho_C + \varphi_R \rho_R,$$

where $\varphi_M, \varphi_C, \varphi_R$ are the filling fraction of the matrix, the coat and the core respectively, and $\rho_M, \rho_C, \rho_R$ are the mass density of corresponding components. The static effective bulk modulus can be obtained as follows:

$$K_E = K_M + \frac{(\varphi_R + \varphi_C)(K_E - K_M)}{1 + 3\varphi_M(\frac{K_E - K_M}{3K_M + 4\mu_M})},$$

with

$$K'_E = K_C + \frac{\varphi_R(K_R - K_C)}{\varphi_R + \varphi_C + 3\varphi_M(K_R - K_C)/(3K_C + 4\mu_C)}.$$

where $K_M, K_C, K_R$ are the bulk modulus of the matrix, the coat and the core, and $\mu_M, \mu_C, \mu_R$ are their shear modulus. They are related to sound speeds of material by $\kappa = \rho c_L^2$ and $\mu = \rho c_S^2$. $c_L$ and $c_S$ are the speeds of longitudinal and shear waves in the material, and $\rho$ is the mass density. Since the speeds of longitudinal or shear waves are frequently mentioned in this paper, we will use the term ‘longitudinal (or shear) velocity’ for short. In addition, the static effective shear modulus can also be evaluated by the following formulae:

$$\mu_E = \mu_M + \frac{5(\varphi_R + \varphi_C)K_M(\mu_M - \mu_M)}{5\mu_M + 6\mu_M(\mu_M - \mu_M)(K_M + 2\mu_M)/(3K_M + 4\mu_M)}.$$

with

$$\mu'_E = \mu_C + \frac{\varphi_R K_R(K_R - \mu_C)}{5(\varphi_R + \varphi_C)K_M(\mu_M - \mu_M) + 6\varphi_M K_C(K_C - \mu_C)(K_C + 2\mu_C)/(3K_C + 4\mu_C)}.$$

Based on Eqs. (1)–(5), the static effective speed of the longitudinal wave ($c_{EL}$) can be calculated as follows:

$$c_{EL} = \sqrt{(K_E + 4\mu_E)/\rho_E}.$$

Furthermore, the non-dimensional starting frequency of Bragg scattering for longitudinal waves in the composite can be evaluated as follows:

$$f_{BR} = \frac{c}{2L}.$$
\[ \Omega_{BG} = \frac{2f_{BG}L}{C_{CL}} = \frac{2L_{CL}}{C_{CL}} = \frac{c_{EL}}{C_{CL}}. \]  

(7)

For instance, by substituting materials’ properties of Case I into Eqs. (6) and (7), we can obtain \( c_{CL} \) as 1192 m s\(^{-1}\), which leads to \( \Omega_{BG} \approx 0.77 \). This evaluation substantially matches the starting frequency of Bragg scattering (see \( \Omega = 0.807 \) in Fig. 2a). The discrepancy of the estimation is caused by the difference between the static and the dynamic effective sound speeds. Although the dynamic effective parameters fluctuate with frequencies, their average values are still approximately equal to the static values. Hence, we use the static effective parameters to estimate the sound speed of the composite \( (c_{E}) \), which further indicates the starting frequency of Bragg scattering effect \( (\Omega_{BG}) \). Base on this idea, we discuss the influence of several parameters on \( c_{E} \) and \( \Omega_{BG} \), such as the shear velocity of the matrix, the lattice constant and the loss factor of the coat.

3.1. Influence of the shear velocity of the matrix on the effective sound speed

Fig. 4a shows the contour map of the effective longitudinal velocity \( \left( c_{EL} \right) \) as a function of the longitudinal velocity of the coat \( \left( c_{CL} \right) \) and the shear velocity of the matrix \( \left( c_{MS} \right) \). Both \( c_{CL} \) and \( c_{MS} \) are varied from 100 m s\(^{-1}\) to 900 m s\(^{-1}\) while other parameters remain the same as Case I. It is found that \( c_{CL} \) does not reduce to 500 m s\(^{-1}\) unless both \( c_{CL} \) and \( c_{MS} \) are smaller than 200 m s\(^{-1}\). This implies that the soft coat of LRs can dramatically reduce the effective sound speed under the precondition that the matrix is a low shear-velocity material. A similar result is found for the effective speed of shear waves in the composite, but we do not discuss it in detail since the shear wave modes are not stimulated for the cases considered in this paper. Fig. 4b shows the starting frequency of the Bragg scattering for longitudinal waves \( (\Omega_{BG}) \) as a function of \( c_{CL} \). The black solid curves represent the variation of \( \Omega_{BG} \) when \( c_{MS} = 800 \) m s\(^{-1}\), and red dashed curves represent \( c_{MS} = 200 \) m s\(^{-1}\).

It is found from the estimations of Eq. (7) that \( \Omega_{BG} \) reduces more rapidly with the decreasing of \( c_{CL} \) when \( c_{MS} = 200 \) m s\(^{-1}\) (the red dashed curve marked with crosses). The same conclusion can be drawn from the results calculated by LMS method, which are marked with circles.

3.2. Influence of the lattice constant on the effective sound speed

From Eqs. (1)–(5), one can find that the filling fractions of components relate closely to \( c_{CL} \). As for Case I, the filling fractions of the matrix, the coat and the core are 84.49\%, 7.57\% and 7.94\%. Evidently, the matrix possesses most volume of the composite. In order to investigate the influence of filling fraction on \( c_{CL} \) and \( \Omega_{BG} \), we vary the lattice constant \( (A) \) from 10 mm to 20 mm and the longitudinal velocity of the coat \( (c_{CL}) \) from 100 m s\(^{-1}\) to 900 m s\(^{-1}\) simultaneously, which is shown in Fig. 5a. The filling fraction of the matrix varies from 73.82\% to 93.46\% with the increasing of \( A \). All the other parameters remain the same as Case I.

As shown in Fig. 5a, \( c_{CL} \) decreases gradually with the decreasing of \( A \) or \( c_{CL} \). All the contour lines are substantially straight and parallel to each other. This implies that the decreasing of lattice constant reduces \( c_{CL} \) independently (without reducing \( c_{CL} \)), but it does not intensify the variation slope of the effective sound speed against \( c_{CL} \). Besides, the reduction of \( c_{CL} \) caused by lattice constant is less obvious than that of the shear velocity of the matrix \( (c_{MS}) \). Fig. 5b shows the starting frequencies of Bragg gaps \( (\Omega_{BG}) \) for two different cases as functions of \( c_{CL} \), where the black solid curves illustrate \( \Omega_{BG} \) for \( A = 15 \) mm, and the red dashed curves illustrate \( A = 10 \) mm. Both the estimation (crosses marked) and the numerical result (circles marked) indicate that the value of \( \Omega_{BG} \) for \( A = 10 \) mm is generally smaller than that for \( A = 15 \) mm. However, the variation slopes of \( \Omega_{BG} \) against \( c_{CL} \) for both cases are approximately the same. Such influence is different from that of the shear velocity of the matrix which affects the slope of \( \Omega_{BG} \) instead of its value directly.

3.3. Influence of the loss factor of the coat on the effective sound speed

Although all the materials are lossless in the above analysis, damping may potentially have an influence on the acoustic behaviour of the LRSM. Fig. 6 shows the real (a) and imaginary (b) parts of the effective longitudinal velocity as a function of the longitudinal velocity \( (c_{CL}) \) and the loss factor \( (\eta_{BC}) \) of the coat. It can be observed that the contour lines in Fig. 6a are almost straight and parallel to the x-axis of the subplot. The real part of \( c_{CL} \) reduces gradually with the decreasing of \( \eta_{BC} \). The variation of \( \eta_{BC} \) has little effect on \( \text{Re}[c_{CL}] \). Hence, the starting frequency of the Bragg frequency of the Bragg

![Fig. 4.](image-url)
scattering effect is also independent of the loss factor of the coat. However, the loss of the coat will provide dissipation when waves propagate in the composite. As shown in Fig. 6b, \( \text{Im}[c_{CL}] \) will increase gradually with the increasing of \( g_C \) and \( c_{CL} \), but the maximum imaginary part is still much smaller than the real part of the effective longitudinal velocity. This implies that the absorbing performance of the coat is poor without the resonance and Bragg scattering effects.

Other parameters of the composite can also affect the effective sound speed of the composite considerably, such as the longitudinal velocity of the matrix, the thickness of the coat and the radius of the core. However, the soft coat of LRs can significantly reduce the effective sound speeds of the composite when the matrix is a low shear-velocity material.

4. Investigation of acoustic behaviours and the coupling of resonance and Bragg scattering effects in LRSMs

From the above discussion, it has been demonstrated that local resonators can prominently reduce the effective sound speed, and hence pull down the frequency region of Bragg scattering. This makes it possible to couple the resonance and Bragg scattering effects together in the sub-wavelength region. Therefore, we redesigned the configuration of LRSM. The same basic LRSM is considered as the configuration of Case I, except that the shear velocity of matrix (\( c_{MS} \)) is set as 200 m s\(^{-1}\) and the lattice constant (\( A \)) as 10 mm, in order to obtain a Bragg gap closed to a resonance gap. Such an LRSM is named Case II (see Table 2).

4.1. Acoustic behaviours of resonance and Bragg scattering effects in LRSMs

Fig. 7a and b show the complex band structure of Case II calculated by the LMS method [32]. The band gap in the frequency region [0.17,0.24] is produced by the local resonance, since the real part of the Bloch wave vector (see Fig. 7a) jumps from \( p \) to 0 at \( X = 0.22 \) inside the gap, and its smallest imaginary part (see Fig. 7b) has a cusp at the corresponding frequency. In contrast, there are three band gaps in the region of [0.28,0.38], [0.41,0.44] and [0.50,0.52]. Each trajectory of \( \text{Re}[K_{CL}] \) in these forbidden bands is either pinned to 0 or \( p \). This
behaviour indicates these regions satisfy the condition of Bragg scattering that the structural periodicity is multiple of the half-wavelength of scattering waves. Since band gaps are the intersection of forbidden bands of both modes, the imaginary part of the Bloch wave vector is a combined result of both wave modes. If there is no alternation of wave modes at two boundaries of a band gap, the imaginary part will exhibit a smooth variation through the gap region, such as the band gap in [0.41, 0.44]. Otherwise, an abrupt change of the slope of the imaginary part will exist in the band gap, such as \( \Omega = 0.51 \). Furthermore, the real part may also exhibits a \( \pi \) jump at the corresponding frequency, if the phases of two gap-boundaries differ from each other by \( \pi \), for instance \( \Omega = 0.38 \).

In addition, the dispersion curves are much more intensive when \( \Omega > 0.51 \) than those in the lower frequency range. This is because the wavelength is comparable with the spatial periodicities in \( x \)- and \( y \)-directions in this frequency region. Thus, oblique scattered waves can provide a strong interference and travel through the composite efficiently. However, all these oblique-scattered bands are inactive for the finite composite with a plane wave normally incident on its boundary from the water.

Fig. 7c shows the logarithmic transmission spectra of Case II calculated by LMS method and FEM. It is observed that the numerical result (blue solid curve) accurately agrees with the simulated result (black dashed curve). The profile of the transmission curve is analogous to the imaginary part of the Bloch wave in the region of [0.17, 0.24] which is the resonance band gap. In contrast, the transmission curve varies smoothly in the region of [0.27, 0.54], which coincides with the longitudinal forbidden band induced by Bragg scattering effect, except two oscillations at \( \Omega = 0.45 \) and 0.53. These oscillations are caused by certain modes of resonance. It is worth noting that there are two considerable transmission dips in the region of [0.24, 0.27], which corresponds to the longitudinal pass band between two band gaps. Such attenuations are produced by the constructive interference of the reflected waves from different boundaries of the finite-thickness composite, which is known as Fabry-Pérot oscillation. Although the resonance and Bragg gaps for Case II are not coupled yet, the enhancement of both effects can already be detected. By comparing with the transmission spectrum of Case I, we find that not only the bandwidth of the resonance gap is broadened, but also the depth of the Bragg gap is enhanced for Case II. Such improvement is caused by the approaching of the resonant frequency to the starting frequency of the Bragg gap, which strengthens the interaction between LRs resonant modes and scattering fields in the matrix. To prove the above analysis, we further investigate the wave fields of the composite simulated by COMSOL at several typical frequencies (see Fig. 8).

Fig. 8a shows the wave field of Case II in the \( yz \)-plane at the lower bounding frequency of the resonance gap (\( \Omega = 0.17 \)). It can be seen from the arrows that a plane wave coming from the left boundary of the matrix inverts its phase after passing through each resonator. The movements of resonators are always opposite to the phases of incoming waves at the left side of each resonator in the matrix. This indicates that the dynamic effective sound speed is reduced by resonators to the value that the half-wavelength of sound is equal to the layer thickness. Therefore, multiple scattering waves in the composite can interact with vibrations of LRs, which eventually broaden the bandwidth of the resonance gap. In contrast, all the arrows in the matrix point to the right side, whereas those in resonators point to the left side as shown in Fig. 8b. This subplot demonstrates the wave field of Case II at the upper bounding frequency of the resonance gap (\( \Omega = 0.24 \)). It implies that the wavelength of the sound in the matrix is much larger than the layer thickness, and resonators vibrate oppositely to the phase of waves in the matrix.

Fig. 8c shows the wave field of the composite when \( \Omega = 0.22 \), which is the pinnacle of the resonance gap. It can be observed that most part of the incident wave is reflected back when it encounters the first resonator. Of course, such efficient attenuation of transmitted waves is caused by the fierce anti-vibrations of LRs around their natural frequencies. However, we can also ascribe the peak attenuation to the superposition of the wave fields of the lower and upper bounding frequencies, since the phase of waves in the composite has a transition at the pinnacle frequency of the resonance gap. One can easily discover that the transmitted waves through the first resonator are just out of phase to each other for the two bounding frequencies (see Fig. 8a and b). This leads to a cancel out of the transmitted waves in the composite, which consequently provides the peak attenuation of transmitted waves.

In contrast, the incoming waves vanishes gradually while propagating through the composite when the frequency lies in the Bragg gap (\( \Omega = 0.41 \)) as shown in Fig. 8d. Although the major displacements of the wave field are still located in the matrix and coats, which are similar to the wave field of Case I (see Fig. 3b), the cores begin to vibrate against the incoming wave. Such vibration is caused by the interaction between LRs’ resonant modes and scattering waves in the matrix, which is also the reason for the enhancement of the Bragg gap. Besides, the half-wavelength of sound in the composite is equal to the layer thickness. This is a necessary condition for the first order of Bragg scattering effect so that the reflected waves from different layers of LRs can interfere constructively.

In addition, Fig. 8e shows the wave field of the composite at the oscillation of the transmission spectrum inside the Bragg gap (see \( \Omega = 0.45 \) and \( \log_{10} T = -8.5 \) in Fig. 7c). The deformation of the composite is mainly concentrated in the matrix at the left side of the

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**Fig. 7.** The real (a) and imaginary (b) parts of the complex band structure of the infinite crystal for Case II, and the logarithmic transmission spectrum (c) of the corresponding finite composite. All the subplots share the same scale of the non-dimensional frequency (\( \Omega \)) as their \( y \)-axes. The \( x \)-axes of subplots (a) and (b) indicate the real and imaginary parts of the \( z \)-component of the reduced Bloch wave vector (\( K/L \)), and the \( x \)-axis of subplot (c) represents the logarithmic transmission coefficient (\( \log_{10} T \)) of the composite. In subplot (a), the blue solid and black dashed lines indicate the longitudinal and shear dispersion curves respectively; the blue dotted lines indicate deformation of longitudinal waves; the shaded areas imply the frequency regions of band gaps; the red solid lines inside gap regions indicate the trajectory of Re\((L/K)\). The blue solid curve in subplot (b) represents the smallest imaginary part of the Bloch wave vector in band gaps. The blue solid curve in subplot (c) indicates the transmission spectrum calculated by LMS method, and the black dashed curve by FEM. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
first resonator, with the maximum displacement located at the centre of the incident boundary. All the LRs and the inner matrix of the composite are motionless. A further investigation shows that the oscillation of the logarithmic transmission coefficient will disappear rapidly if the matrix contains any damping. In fact, this is a resonant mode of the surface wave, which appears within the longitudinal forbidden band. Such a surface resonant mode is known as the surface-localized mode [34,35]. Obviously, this resonant mode can only exist in the finite-thickness composite. Hence, no corresponding dispersion curve is found in the complex band structure of the infinite crystal (see Fig. 7a). Moreover, the oscillation of the transmission spectrum that occurs at $\Omega = 0.53$ is induced by a similar surface-localized mode.

### 4.2. The coupling of resonance and Bragg scattering effects in LRSMs

Based on Case II (listed in Table 2), several parameters have been investigated individually in order to couple resonance and Bragg scattering effects. Fig. 9 shows the logarithmic transmission coefficient ($\log_{10} T$) as a function of the shear velocity of the coat ($c_{CS}$) and the non-dimensional frequency ($\Omega$). The variation of $c_{CS}$ is discussed exclusively here because it will tune the natural frequency of LRs without changing the Bragg scattering of longitudinal waves. Particularly, for the original Case II ($c_{CS} = 83$ m s$^{-1}$), the first transmission gap with a maximum attenuation inside is caused by resonance, whereas the gap in the higher frequency range is produced by Bragg scattering (see Fig. 9).

As shown in Fig. 9, there is a black straight line in the transmission gap, which ranges from $\Omega = 0.22$ when $c_{CS} = 80$ m s$^{-1}$ to $\Omega = 0.35$ when $c_{CS} = 150$ m s$^{-1}$. This straight line indicates the variation of the natural frequency of LRs, since the black colour in the hot-scale represents the maximum attenuation of the transmission gap. It is observed that the resonance gap keeps approaching the Bragg gap, when the shear velocity of coat is smaller than 122 m s$^{-1}$ and increases gradually. Meanwhile, the lower bounding frequency of the Bragg gap remains constant while the upper bounding frequency increases continuously. Most importantly, the approaching of two transmission gaps does not only broaden the bandwidth of the resonance gap, but also enhances the depth of the Bragg gap. When $c_{CS} = 122$ m s$^{-1}$, the natural frequency of LRs coincides with the lower bounding frequency of the Bragg gap. Hence the resonance and Bragg gaps are coupled together, and the composite exhibits a broad transmission gap in the region of $[0.19, 0.54]$. If the shear velocity of the coat keeps increasing, the coupled gaps will separate from each other again with their locations exchanged, which can be verified by the switch of the maximum attenuation to the upper gap. For these cases ($c_{CS} > 122$ m s$^{-1}$), the upper bounding frequency of the Bragg gap stays constant ($\Omega = 0.28$) at the same frequency as the lower bounding frequency was before the gaps’ coupling, and the bandwidths of two gaps keep shrinking.
Most of the phenomena observed here are similar to the gap-coupling behaviours reported in Ref. [28], except for two differences which are caused by the characteristic of the finite-thickness composite. Firstly, the oscillations of the transmission spectra at \( \Omega = 0.45 \) and 0.53 for Case II do not shift their positions with the variation of \( C_C \). Such phenomenon verifies that these oscillations are caused by surface-localized modes instead of resonant modes of LRs, since they are irrelevant to the properties of coat. Secondly, considerable attenuations are found in the region of the longitudinal pass band which exists between two band gaps. Such attenuations are the extension of Fabry-Pérot oscillations for different configurations of LRSMs.

Fig. 10 shows the complex band structure and the logarithmic transmission spectrum of the LRSM whose gaps are coupled together (when \( C_S = 122 \text{ m s}^{-1} \)). This gap-coupled LRSM is named Case III (see Table 2). It can be observed from Fig. 10a that the longitudinal wave mode exhibits a broad forbidden band in the frequency region of [0.19,0.56] due to the coupling of resonance and Bragg scattering effects. However, this forbidden band is divided into several narrow band gaps by the shear dispersion curves as shown in Fig. 10b. Nevertheless, the log-scaled transmission spectrum exhibits an integrated attenuation gap in the frequency region corresponding to the longitudinal forbidden band, which is shown in Fig. 10c. In this subplot, the transmission spectrum calculated by FEM (black dashed curve) accurately agrees with the counterpart calculated by LMS method (blue solid curve). Hence, a further investigation of the wave fields inside the composite is carried out according to the FEM’s results.

Firstly, the transmission gap has a pinnacle at \( \Omega = 0.29 \). When the propagating sound just lies in this frequency, the lossless resonators will vibrate fiercely, and the composite will reflect the incoming wave efficiently. The wave field at this pinnacle is similar to the wave field shown in Fig. 8c. Secondly, for other frequency inside the transmission gap except the pinnacle (\( \Omega = 0.29 \)) and the oscillations (\( \Omega = 0.45 \) or 0.53), the wave field of Case III is generally analogous to Fig. 8a. In detail, one half-wavelength of the propagating sound in the matrix is equal to the layer thickness; the resonators interact strongly with the multiple scattering waves in the matrix while its phase is opposite to the phase of the incoming wave; and the incoming wave attenuates gradually during its propagation inside the composite. These phenomena imply that each resonator reflects the incoming wave efficiently, and the reflected waves from these resonators interfere constructively, which eventually yields a prominent sound blocking performance in the coupled gap. Hence, by tuning the natural frequency of LRs to the lower bounding frequency of the Bragg scattering effect, we can strengthen the interaction between the vibrations of resonators and the scattering waves in the matrix, thus broaden the bandwidth and enhance the depth of the transmission gap simultaneously.

5. Discussion of the sound blocking and absorbing performance of LRSMs with damped coats

Although the loss factor of coat (\( \eta_C \)) does not affect the effective sound speed obviously as discussed in Section 3.3, it does affect the sound blocking and absorbing performance of the LRSM. Fig. 11 shows the transmission and absorption spectra of the finite composite for Case I and III when \( \eta_C \) equals 0, 0.05, 0.1 and 0.2 respectively. All these spectra are calculated by the LMS method. As shown in Fig. 11a, the pinnacle of the resonance gap for Case I in the region of [0.17,0.22] weakens rapidly with the increasing of \( \eta_C \) (\( \log_{10} \eta_T \) varies from \(-10.6 \) to \(-1.7 \)), whereas the Bragg gap in the region of [0.75,1.04] remains the same. Similar behaviours are also observed in Case III: the increasing of \( \eta_C \) whittles away the pinnacle of the transmission gap at \( \Omega = 0.29 \), but does not affect the coupled gap in other regions, which is shown in Fig. 11b. Since Case III has a strong interaction between resonators and waves in the matrix, the sound blocking performance does not only depend on the anti-phase vibration of LRs, but also on the interference of scattering waves. Therefore, the pinnacle for Case III shrinks more slowly than that for Case I when \( \eta_C \) increases gradually (\( \log_{10} \eta_T \) varies from \(-9.8 \) to \(-5.1 \) at the pinnacle of the coupled gap). In addition, the increasing of \( \eta_C \) also provides considerable attenuation in the original pass band, which eventually broadens the bandwidth of the coupled transmission gap. Hence, the gap-coupled case (Case III) does not only obtain a deeper and broader transmission gap than the initial case does (Case I), but also maintains a good sound blocking performance even if the coats of LRs include considerable loss.

Fig. 11c and d show the absorptions of two cases under different conditions of \( \eta_C \). With the increasing of \( \eta_C \), both cases enhance their sound absorbing performances gradually in the low frequency region (\( \Omega < 0.3 \)). By comparing the absorptions of two cases with the same loss factor of the coat, we can conclude that the absorbing performance of Case III (see Fig. 11d) is generally better than the counterpart of Case I (see Fig. 11c). The improvement of the absorption for Case III is mainly attributed to the rearrangement of the lattice constant and the shear velocity of the matrix, which sufficiently reduce the effective sound speed of the composite. According to Eq. (6), we can obtain the static effective speed of longitudinal waves (\( c_{el} \)) as 1192 m s\(^{-1}\) and 440.6 m s\(^{-1}\) for Case I and III respectively. On account of the reduction of \( c_{el} \), the destructive interference of the reflected waves from two boundaries of Case III arises at a lower frequency than that of Case I. This
Fig. 11. The log-scaled transmission spectra for Case I (a) and III (b); the absorption performance for Case I (c) and III (d); (e) the linear-scaled transmission spectra of two cases with lossless coats and the absorption spectra with $g_C = 0.1$. All the subplots apply the non-dimensional frequency ($\Omega$) as their x-axes; the transmission spectra (a) and (b) share the same scale of $\log_{10} T$ as their y-axes; the absorption spectra (c) and (d) share the same absorption coefficient ($A$) as their y-axes, which is the energy ratio of the total dissipation to the incoming wave. The y-axis of subplot (e) indicates both linear-scaled transmission ($T$) and absorption ($A$) coefficients. The blue solid, black dotted, red dashed and green dash-dot curves in subplot (a–d) represent the spectra of LRSMs when the loss factor of the coat ($g_C$) is 0, 0.05, 0.1 and 0.2 respectively. They share the same legend which is shown in subplot (c). The blue solid lines in subplot (e) indicate the transmission spectra of two cases with lossless coats, and the red dashed lines indicate their absorption with $g_C = 0.1$. Besides, the curves with crosses represent spectra of Case I, while those with circles represent spectra of Case III. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 12. (a) The wave field in the yz-plane at the first destructive interference of reflected waves of Case III ($\Omega = 0.07$); (b) at the second destructive interference of Case III ($\Omega = 0.13$); (c) at the third destructive interference of Case III ($\Omega = 0.18$); and (d) at the first destructive interference of Case I ($\Omega = 0.15$). The incident wave comes from the left side of the composite. The hot-scaled image indicates the normalized displacement of the wave field with red meaning maximum displacement and white meaning no movement. The black arrows on the image indicate the direction and amplitude of the displacement at each point.
interference is a typical phenomenon of the Fabry–Pérot oscillation. As shown in Fig. 11e, the destructive interference of reflections for Case III (the blue solid line with circles) arise when \( \Omega = 0.07, 0.13 \) and 0.18, which can be identified by the occurrence of the maximum transmission (\( T = 1 \)). In contrast, the first destructive interference for Case I arises at \( \Omega = 0.15 \) (the blue solid line with crosses), which is already close to the lower bounding frequency of the resonance gap, and the other two occur at even higher frequencies (\( \Omega = 0.171 \) and 0.175).

Fig. 12a–c show the wave fields of Case III with lossless coats when \( \Omega = 0.07, 0.13 \) and 0.18. It can be recognised from Fig. 12a that the half-wavelength of the sound in the composite is just equal to the total thickness of the finite composite for Case III when the first maximum transmission arises (\( \Omega = 0.07 \)). Since the frequency is still far from the natural frequency of resonators, LRs move along with the matrix instead of vibrating oppositely. The corresponding absorption (the red dashed line with circles) is 0.18 at this frequency as shown in Fig. 11e. Similarly, the total thickness of the composite is just equal to or twice of the wavelength of the sound when \( \Omega = 0.13 \) or 0.18 respectively, which is shown in Fig. 12b and c. It is worth noting that the forced vibrations of resonators become more and more fiercely with the frequency approaching the natural frequency of LRs. Meanwhile, the corresponding absorptions increase to 0.38 and 0.73 (see Fig. 11e) for \( \Omega = 0.13 \) and 0.18 respectively.

In contrast, the half-wavelength of the sound keeps longer than the total thickness of the finite composite for Case I until \( \Omega = 0.15 \) (see Fig. 12d). Besides, the vibrations of LRs are more prominent than the multiple scattering waves in the composite at this frequency, since it is already close to the natural frequency of LRs. The absorption for Case I at this frequency is 0.36 (see the red dashed line with crosses in Fig. 11e). Through the analysis of these wave fields, we conclude that Case I can only absorb the sound efficiently by the forced vibration of LRs when the frequency is close to their natural frequencies, whereas Case III can also absorb the low-frequency sound by the multiple scattering effect in the composite. Since the wavelength of the sound for Case III is shorter than that for Case I at the same frequency, the damped coats of Case III are more efficient at dissipating the sound energy than those of Case I. Consequently, the gap-coupled case (Case III) exhibits a better absorbing performance in the low frequency region than the initial case (Case I).

6. Conclusion

By using the effective medium theory, we analysed the effective acoustic properties of the three-dimensional locally resonant sonic material (LRSM). The result showed that the effective sound speed of the composite could be significantly reduced by simultaneously slowing down the longitudinal velocity of the coat and the shear velocity of the matrix, as well as by compressing the lattice constant. This consequently pulled down the frequency region of the Bragg scattering effect. By comparing the complex band structure with the log-scaled transmission spectrum of the LRSM, we found that the transmission gap of the resonance had an analogous profile with the smallest imaginary part of the corresponding band gap, whereas the Bragg transmission gap coincided with the frequency region of the longitudinal forbidden band induced by Bragg scattering. The transmission spectra calculated by the layer-multiple-scattering method agreed well with those by COMSOL for all the cases considered. Therefore, we further analysed the wave fields of LRSMs at the boundaries, the pinnacle of the resonance gap and the mid-frequency of the Bragg gap. It has been found that the wave field for the pinnacle of the resonance gap was a superposition of wave fields for two gap-boundaries. As for the Bragg gap, the half-wavelength of the sound in the composite was equal to the layer thickness so that the reflections from different layers of resonators interfered constructively. In addition, the transmission spectra displayed the surface-localized modes and the Fabry–Pérot oscillations, which are the characteristic behaviours of a finite-thickness material immersed in water ambience. By varying the shear velocity of the coat, we investigated the coupling behaviour of two kinds of transmission gaps. The analysis revealed that the approaching of two gaps not only broadened the bandwidth of resonance gap, but also enhanced the depth of Bragg scattering gap. When the two transmission gaps were coupled together, the composite exhibited a broad sound blocking performance in the sub-wavelength region. A further discussion showed that the coupled gap could maintain a good sound blocking performance even if coats included considerable loss. Besides, the coupled gap had a better absorbing performance in the low frequency region than the uncoupled one. This work offered a new technique to enhance the sub-wavelength sound blocking and absorbing performance of 3D LRSMs, which could be potentially used for underwater sound insulation and noise control purposes.

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