Back ing effects on the underwater acoustic absorption of a viscoelastic slab with locally resonant scatterers

Honggang Zhao *, Jihong Wen, Haibin Yang, Linmei Lv, Xisen Wen

Vibration and Acoustics Research Group, Science and Technology on Integrated Logistics Support Laboratory, College of Mechatronic Engineering and Automation, National University of Defense Technology, Changsha, Hunan 410073, PR China

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A B S T R A C T
Back ing effects on the underwater acoustic absorption of a viscoelastic polymer slab embedded with locally resonant scatterers are reported. The polymer slab is embedded with two layers of locally resonant scatterers, i.e. Al spheres coated by soft silicon rubber. Theoretical absorption coefficients of the polymer slab under different backings using a layer multiple scattering method show good agreement with the experimental results, which supports unambiguously the experimental observation. Then relations between the resonance modes and the low-frequency absorption peaks of the composite slab are clarified to address the absorption mechanisms. It shows that the mass of the steel backing affects evidently the low-frequency absorption, the absorption peak shifts to lower frequency range while increasing the backing mass.

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1. Introduction

Locally resonant sonic materials (LRSMs) are generally known as composite epoxy embedded with various heavy spheres coated by soft rubber with periodic or random position [1]. It has been shown that the soft coating layer and the heavy core are responsible for a band gap at low frequency about two orders of magnitude lower than that predicted by Bragg’s condition. For practical applications, much effort has focused on air sound isolation induced by the gaps of different LRSMs [2]. Our investigations in acoustic dissipation by LRSMs have shown the low frequency acoustic absorption while considering the damping between the components [3,4]. Motivated by understanding of the energy dissipation in LRSMs, the locally resonant scatterers (LRSs) are introduced to improve the low-frequency acoustic absorption of the water impedance-matched polymer. Experimental measurement for acoustic absorption of viscoelastic polymer slabs embedded with LRSs has been reported [5]. The fundamental mechanism operating in the localized resonance for acoustic absorption has been investigated by referring the mode conversions during the Mie scattering of a single scatterer. Further, the shape of the scatterer has also been taken into account [6].

In fact, viscoelastic polymers with various shapes of cavities [7–14] or solid scatterers [15,16] are widely used in modern underwater acoustic applications. In most cases, scattering resonances including monopole and dipole types have been used to enhance the low frequency absorption of viscoelastic composites. The monopole resonance may be induced by various cavities [7–14] and the dipole type may occur in case of heavy spheres [5,6,15,16]. For spherical scatterers, the absorption properties can be investigated conveniently by the layer multiple scattering method (LMSM), which is developed for gap analysis in phononic crystals [17,18]. The computation is simplified by using the Bloch theory. In comparison with the finite element method [9–11], although the flexibility in model is limited, there are two important advantages of the LMSM, one is the fast computation, and the other is the convenient understanding of the physical absorption mechanism.

For actual applications, the design of viscoelastic polymers embedded with locally resonant scatterers must consider the effect of the backing panel, which is not considered in our previous works [5,6]. The effects of the steel backing on the absorption are shown by the experimental and theoretical results in present paper. To address the absorption mechanisms, the relation of the resonance mode and the absorption peak of the composite slab is clarified based on the theoretical and experimental results. For comparisons, other backings are also considered.

2. Experiment

2.1. Model and sample

Fig. 1a shows a sketch model of an absorption structure including a sample slab and a backing under the Cartesian coordinates...
system. The whole structure can be considered as a sequence of different layers perpendicular to Z-axis. The sample is made of viscoelastic polymer embedded with two layers of LRSs. The interlayer positions of the scatterers are denoted by \( L_i \) (\( i = 1, 2, 3 \)), where \( L_i \) denotes the center of the first layer of LRSs from the surface of the sample, \( L_2 \) is the distance between both the layers of LRSs, and \( L_3 \) denotes the space between the second layer of LRSs and the other surface of the sample. The thickness of the whole sample is 55 mm, and \( L_1 = L_3 = 13.75 \) mm, \( L_2 = 27.5 \) mm. Considering the limitation of the experiment apparatus, i.e. an impedance tube with an inner diameter \( d = 120 \) mm, the sample has an overall shape of short cylinder, which possesses a total diameter \( D = 118 \) mm. Each layer contains 30 scatterers, which are arranged in a quasi-triangular lattice with a mean adjacent distance 19.5 mm. The position of the scatterers is shown in Fig. 1b. Each LRS has a uniform spherical shape, in which a core sphere (radius \( r \)) coated by soft silicon rubber (with outer radius \( r_1 \)), here \( r = 5 \) mm, and \( r_1 = 7.5 \) mm. In order to reduce the weight of the sample and facilitate the understanding of the resonance mode of the backing, we choose Al material as the core (which separates the resonances induced by the backing and the LRS). The soft coating is made of a type of two-component, room temperature vulcanizing silicone rubbers (RTV-2). It is available in a variety of hardness and suitable for various mold techniques. To begin, the core is coated by the silicon rubber using a self-made mold. Then the sample is fabricated by a hollow cylinder mold with an inner diameter \( D = 118 \) mm in the following procedure:

1. A polymer slab with thickness of \((L_1 - r_1)\) is made.
2. With a proper interval time, the coated scatterers are arranged on the surface of the polymer slab according to the design, and the rest space of the mold with a height \((L_1 + L_2 - r_1)\) is filled by the polymer.
3. The multi-layer sample is fabricated by an iterative procedure (2) according to \( L_i \).

2.2 Absorption measurement method

Measurement of the absorption coefficient is conducted in a standard impedance tube apparatus. The steel tube has an inner (outer) diameter 120 mm (180 mm), and 5 m long. The schematic of the apparatus is showed in Fig. 2. The impedance tube is fully filled with water. A plane wave incidence is excited at one end of the tube while the sample is positioned at the other end, where various backings can be attached on the surface of the sample.

Under normal incidence, unlike the backing, the impedance tube almost does not interact with the testing waves because the first order resonant frequency of the tube walls is higher than that of the upper cut-off frequency in the present test.

The experimental approach is based on a frequency response function method (FRFM) using two microphones [19]. One can get the frequency response function \( H_{12} = P_2/P_1 \), \( P_1 \) and \( P_2 \) denoting the acoustic pressure measured by Micr. 1 and Micr. 2 respectively. The complex pressure reflection coefficient \( (R) \) is given by:

\[
R = \frac{H_{12} e^{jkL} - e^{jk(L+s)}}{e^{-jk(L+s)} - H_{12} e^{jkL}},
\]

where \( k \) is the wave number, defined as \( \omega/c \), and \( c \) is the acoustic velocity of the water. \( L \) denotes the distance between the Micr. 1 and the surface of the sample, and \( s \) is the spacing between two microphones. Considering the fully reflective termination in the experiment, the sound energy absorption coefficient \( (\alpha) \) can be calculated from the following equation:

\[
\alpha = 1 - |R|^2.
\]

3. Discussion

3.1 Comparison of theoretical and experimental results

Generally, the steel backing can be considered as a finite steel slab followed by half infinite air. In order to understand the variation and the formation of the acoustic absorption of the sample under the steel backing, we compare the acoustic absorption of the sample under different backings in the experiment. We choose the backings as half infinite air (air backing in the following), steel
backings with different thickness (5 mm and 10 mm respectively). Fig. 3 gives the experimental absorption coefficients (labeled by E-) of the sample under different backings. One can readily see that an absorption peak appears under each backing. When the sample has the air backing, 5 mm and 10 mm steel backings in turn, the absorption peak shifts to lower frequencies and the bandwidth of the absorption becomes slightly narrower at the same time.

To compare with the experimental results, the acoustic absorption of the sample under the corresponding backing is computed by the LMSM. For simplifying the analysis, the composite slab is assumed to be infinitely along intra-layer (XOY) plane, and the scatterers in each layer are arranged in a triangle lattice. The lattice constant, i.e. the nearest distance between two neighbor scatterers, is 19.5 mm. A plane longitudinal wave is incident from the half infinite water (see Fig. 1a). Material parameters for all components are fixed without including the dependence on frequency for simplicity, and are listed in Table 1. The parameters of the polymer and the silicon rubber are measured using a dynamic mechanical analyzer (DMTA IV), and the parameters at 1 kHz are used. The viscosity of other components is neglected and the corresponding damping factor is set to zero. The density of water (air) is 1000 kg m\(^{-3}\) (1.29 kg m\(^{-3}\)) and the longitudinal wave speed is 1480 m s\(^{-1}\) (340 m s\(^{-1}\)). The theoretical results (labeled by T-) are compared with those from the experiment in Fig. 3. One can readily see that the results of the calculation under different backings show good agreement with those of the experiments. Little difference between the theoretical and experimental results exists. For instance, the bandwidth of the absorption peaks by calculation is narrower than those of the experiments. This phenomenon may be induced by small volume of air bubbles (although eliminated by some means, for example, vacuumizing and static pressure) remaining in the actual sample. The bubbles enhance the wave scattering and visco-thermal effects [20,21], then the acoustic absorption in actual experiment. However, the dissipations induced by the bubble scattering and the visco-thermal effects are not considered in the present computation.

### 3.2. Discussions

In order to understand the origin of the above absorption peaks, we also calculate the absorption of the composite slab under water backing, denoted by the long dashed line in Fig 3. One can see that there is weak absorption peak at 2080 Hz. Here the polymer is water impedance-matched, so the water backing affects little the absorption of the composite slab. The weak absorption peak is induced by the weak resonance of the scatterer determined by the light Al core, which can be verified by the analysis of Mie scattering. For a spherical scatterer, the Mie scattering matrix (\(\mathbf{T}\)) for each partial wave of \(l\) order is given by \(\mathbf{T}_l\) [5], in which \(L, M\) and \(N\) denote respectively one longitudinal wave mode and two transverse wave modes. It shows that the \(L\) and \(N\) modes are coupled to each other, while the \(M\) mode is decoupled. Since only longitudinal incidence exists underwater, the \(M\) mode is out of our consideration in following discussions. The meanings of the elements in \(\mathbf{T}_l\) are clear.

### Table 1

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg m(^{-3}))</th>
<th>Young’s modulus (Pa)</th>
<th>Poisson’s ratio ((\nu))</th>
<th>Damping factor (tan(\delta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polymer</td>
<td>1100</td>
<td>2.97 \times 10^3</td>
<td>0.498</td>
<td>0.3</td>
</tr>
<tr>
<td>Silicon</td>
<td>1300</td>
<td>2.0 \times 10^6</td>
<td>0.46</td>
<td>0.3</td>
</tr>
<tr>
<td>Rubber</td>
<td>7890</td>
<td>2.09 \times 10^{11}</td>
<td>0.275</td>
<td>0</td>
</tr>
<tr>
<td>Steel</td>
<td>2690</td>
<td>7.44 \times 10^{10}</td>
<td>0.334</td>
<td>0</td>
</tr>
</tbody>
</table>

For example, \(LL\) stands for the conversion from a longitudinal incidence into a longitudinal outgoing wave during the scattering procedure. Because the localized resonance at low frequency is a dipole type, we focus on the mode conversions during the dipole (\(l = 1\)) scattering. Fig. 4 shows the variation of the absolute values of the \(T_1\) elements for the coated Al sphere in unbounded polymer. From Fig. 4a, one can see that the longitudinal incidence is scattered by the \(LN\) channel exceeding the \(LL\) channel during the dipole resonance, which suggests that the longitudinal incidence prefers to scatter into transverse wave. From Fig. 4b, it shows that the amplitudes of the \(T_1\) elements for the transverse wave incidence present about five orders of those for the longitudinal wave incidence, which hints that the transverse wave dominates the scattering. Generally, the transverse wave damps rapidly in viscoelastic polymer, then the acoustic dissipation is enhanced. Accordingly, the absorption peak of the polymer slab under the water backing appears around the dipole resonant frequency (comparing Figs. 3 and 4). The amplitude of the absorption peak is mainly determined by the amplitude of \(LN\) scattering in \(T_1\) since only longitudinal wave incidence exists underwater.

Under the air backing, however, we can readily see that the absorption coefficient is much enhanced and a new absorption peak comes into being. The acoustic reflection from the polymer/air interface at the backing forms a partial standing wave in the composite slab because of the energy damping, and there exists corresponding standing wave resonance in the composite slab. The standing wave resonance enhances the mode conversion from longitudinal to transverse waves and the multiple scattering among the scatterers, so the energy dissipation in the composite slab is enhanced and a new absorption peak comes into being. From the dotted line in Fig. 3, one can see that this new absorption peak combines that induced by the localized resonance because both absorption peaks have a close position.

For the steel backing, a distinct feature from Fig. 3 is that the absorption peak shifts to lower frequencies while increasing the thickness of the steel slab. The absorption peak induced by the localized resonance appears again because the space between both the absorption peaks increases. However, the absorption peak induced by the localized resonance is smoothed in the experimental results, which can be approached as a linear function leading to the expression:

\[
Z_m = j\Omega Z_s \tan(k_s d),
\]

where \(d\) denotes the thickness of the steel slab, \(Z_s\) is the characteristic impedance of the steel and \(k_s\) the wave number, both parameters being defined as \(Z_s = \rho_s c_s\) and \(k_s = \omega c_s\), here \(\rho_s\) and \(c_s\) denotes the density and longitudinal velocity of the steel.

Note that the low frequency range are focused and \(d\) is less than 10 mm, so the \(k_s d\) is a small argument, the tangent can be approached as a linear function leading to the expression:

\[
Z_m = j\Omega Z_s k_s d = jp_s c_s d.
\]

The above equation demonstrates that the input impedance of the steel backing is not dependent on the velocity of the steel for low values of \(k_s d\), is proportional to the mass \(\rho_s d\). Based on the absorption peak variation with the different steel backing in Fig. 3, the absorption peak shifts to lower frequencies while increasing the mass \((d\) or \(\rho_s\)) of the steel backing. The absorption peak is induced by the standing wave resonance from the overall resonance of the steel backing. The physical mechanism of the resonance...
the resonance of the steel backing shifts to lower frequencies while increasing the backing mass. The resonance enhances the mode conversions from longitudinal waves to transverse waves and the multiple scattering among the scatterers, which dominates the low-frequency acoustic absorption.

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