Two-Dimensional Locally Resonant Phononic Crystals with Binary Structures

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The lumped-mass method is applied to study the propagation of elastic waves in two-dimensional binary periodic systems, i.e., periodic soft rubber/epoxy and vacuum/epoxy composites, for which the conventional methods fail or converge very slowly. A comprehensive study is performed for the two-dimensional binary locally resonant phononic crystals, which are composed of periodic soft rubber cylinders immersed in epoxy host. Numerical simulations predict that subfrequency gaps also appear because of the high contrast of mass density and elastic constant of the soft rubber. The locally resonant mechanism in forming the subfrequency gaps is thoroughly analyzed by studying the two-dimensional model and its quasi-one-dimensional mechanical analog. The rule used to judge whether a resonant mode in the phononic crystals can result in a corresponding subfrequency gap or not is found.

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The propagation of elastic or acoustic wave in periodic heterogeneous materials has received much renewed attention in recent years [1–10]. Because of the periodicity of such structures, there exist frequency ranges in which waves are forbidden, giving rise to phononic band gaps which are analogous to photonic band gaps [11–13] for electromagnetic waves. These new materials can be of real interest because of the rich physics of acoustic and elastic systems, where the wave can have mixed longitudinal and transverse modes, and where a large contrast between the elastic parameters is allowed. For example, locally resonant (LR) phononic crystals (PCs) consisting of very soft rubber [1,4,9] (with an elastic constant of five orders lower than common solids) and other components are most likely to obtain the low-frequency gaps with a structure of small dimensions, which can lead to promising applications such as low-frequency vibration or noise insulations.

The lumped-mass (LM) method [7] is proposed recently by us as a new way to compute the band structure of two-dimensional (2D) PCs. The idea of the LM method is the discretization of the continuous system. Thus, the density of the medium is concentrated on discrete points as particles and the elastic constants are treated as linear elasticity between the adjacent particles. By employing a finite number of particles in one period, the band structure can be calculated numerically with our discrete model. Compared with other techniques, the LM method converges faster and its convergence is independent of the model. Compared with other techniques, the LM method converges faster and its convergence is independent of the model. The first flat branch locates at 1.9 in the reduced frequency unit, corresponding to a lattice vibration mode localized in the softer material (polyethylene).

Here, in order to discover the rule used for judging whether a LR mode can result in a corresponding subfrequency gap or not, we analyze the binary counterparts of the LR PCs which consist of a lattice of soft rubber cylinders in epoxy.

First, we consider two of the same square lattices of soft rubber (the same as Refs. [1,4,9]) and vacuum cylinders in epoxy with a filling fraction $\frac{2}{1.00}$ of the LR PCs which consist of a lattice of polyethylene cylinders in Al were previously researched [6], where no complete gap was observed and where optical-like flat branches were found in the band structure. The first flat branch locates at 1.9 in the reduced frequency unit, corresponding to a lattice vibration mode localized in the softer material (polyethylene).

The existence of gaps in a frequency range of 2 orders of magnitude lower than the one resulting from the Bragg scattering mechanism, and explained the origin of this phenomenon as due to the localized resonances associated with scattering units. Two-dimensional PCs composed of lattices of polyethylene cylinders in Al were previously researched [6], where no complete gap was observed and where optical-like flat branches were found in the band structure. The first flat branch locates at 1.9 in the reduced frequency unit, corresponding to a lattice vibration mode localized in the softer material (polyethylene).

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the original dispersion curves (illustrated in Fig. 2) of the vacuum-epoxy PCs, while the others just thrill through them.

In detail, from Figs. 1(a) and 3, we can observe a subfrequency gap in the frequency range from 0.0046 to 0.0058 (in reduced units), which is 2 orders of magnitude lower than the one resulting from Bragg scattering, as observed in ternary cases [1,4]. This subfrequency gap results from the first resonant mode plotted in Figs. 3(a) and 3(b). At point \( T_{0a} \) [Fig. 3(a)], the amplitude of the vibrations is well concentrated in the region of rubber cylinders, and it is very small in the hosting media. At point \( T_{0b} \) [Fig. 3(b)], the lattice vibrations are almost the same except that the vibrations in the hosting media are notable and in the reverse phase of that in the cylinders. For both cases, rubber cylinders vibrate as mass-spring oscillators. The time harmonic forces from oscillators to the hosting structure split the original dispersion curves, and a narrow gap is generated. As for the second and third resonant modes illustrated in Figs. 3(c) and 3(d), which look like the field map of a dipole, the forces from the rubber oscillator to the hosting structure are counteracted. Their corresponding flat branches in the band structure thrill through the original dispersion curves and no gap is generated.

For the in-plane modes shown in Fig. 1(b), the first resonant mode illustrated in Fig. 4(a) looks like a clockwork torsion spring. Because of the aforesaid reason, i.e., the composition of the forces to the hosting structure contributed by the oscillator is zero, its corresponding flat branches in band structure just thrill through the original dispersion curves and no gap is generated. As for the second and third resonant modes illustrated in Figs. 4(b) and 4(c) where the compositions of forces are not zero, the dispersion curves are cut off at their corresponding eigenfrequencies (0.00988 and 0.00996 in reduced units) in Fig. 1(b).

In order to support the resonant origin of the subfrequency gaps, we plot in Fig. 5 the dependency on \( f_h \) of the band edges determining the lowest gap of the out-of-plane modes. In Figs. 5(a) and 5(b) the lattice constant \( a \) is a constant, while in Fig. 5(c) the radius of the soft rubber cylinder \( r \) keeps constant as 8 mm. The behavior of...
the band edges is completely different from that characterizing a Bragg gap (see, for example, Fig. 2 in Ref. [4] or Fig. 3 in Ref. [9]). The upper panel shows that the reduced frequencies of both edges reduce with $f_h$. This is a signature of their resonant origin. The pinning of the edges are a consequence of their resonant characters of their associated modes. Particularly, the bottom edge corresponds to the localized vibration mode inside the rubber, where the eigenfrequency is determined by the oscillator consisting of rubber only. The upper edge corresponds to the vibration mode where the epoxy is also involved. So the eigenfrequency of the upper edge is slightly higher than the previous one. In Fig. 5(c) where the radius of the rubber cylinders is a constant, the real frequency of the lower edge keeps constant no matter how other parameters such as lattice types or lattice constants change. As a result, band gaps in such a system will also appear even in the absence of periodicity. This is a signature of its localized nature.

In order to understand the physical insight of the complicated system more clearly, we introduce a simple quasi-one-dimensional analog model. As shown in Fig. 6(a), the model consists of a linear beam with oscillators periodically attached to it. The Yang’s module, cross section area, and length density of the beam are $E/0.0136 \times 10^9$ Pa, $S/0.0002 \times 10^{-4} m^2$, and $\rho_i = 2.8$ kg/m, respectively. The lattice parameters are $a_A = 19$ mm and $a_B = 1$ mm. Each oscillator consists of two equal masses ($m_1 = m_2 = 16$ g) and three springs ($k_1 = k_2 = 38,000$ N/m and $k_m = 24,700$ N/m). The parameters are chosen in order to fit the 2D binary LR PCs as for Fig. 1(a).

The mechanical system can be solved analytically by the transfer matrix method [14]. The corresponding band structure is illustrated in Fig. 6(b).

In Figs. 6(b) and 7, we can observe similar characters shown in Figs. 1(a) and 3. A subfrequency gap exists in the same frequency range as appears in Fig. 1(a). This subfrequency gap results from the first resonant mode plotted in Figs. 7(a) and 7(b). At point $B_{0a}$, the vibration of the system is well concentrated on the oscillators where
the two masses vibrate with the same phase and amplitude. So the eigenfrequency of the lower edge of the gap can be calculated with

\[
\frac{a}{2 \pi \epsilon_{\text{beam}}} \sqrt{\frac{k_1 + k_2}{m_1 + m_2}} = 0.0046. \tag{1}
\]

At point \(B_0\) [Fig. 7(b)], the mode is almost the same except that the vibration of the beam is not zero and in reversed phase to that of the masses. Its eigenfrequency that determines the upper edge of the gap can be calculated with

\[
\frac{a}{2 \pi \epsilon_{\text{beam}}} \frac{k_1 + k_2}{m_1 + m_2} \left( \frac{1 + m_1 + m_2}{\rho_1 a} \right) = 0.0058. \tag{2}
\]

For both points \(B_{0a}\) and \(B_{0b}\) [Fig. 7(b)], the time harmonic forces from the oscillators to the beam split the original dispersion curves [of the beam only, illustrated as dashed lines in Fig. 6(b)]. A subfrequency gap is then generated. As for the second resonant mode illustrated in Fig. 7(c) where the two masses vibrate with reversed phases, the forces from the oscillator to the beam are counteracted. Their corresponding flat branches thrill through the original dispersion curves and no gap is generated. Its eigenfrequency can also be calculated with

\[
\frac{a}{2 \pi \epsilon_{\text{beam}}} \sqrt{\frac{k_1 + 2k_m}{m_1}} = 0.007. \tag{3}
\]

In conclusion, we have studied the propagation of elastic waves in two-dimensional binary phononic crystals consisting of lattices of soft rubber cylinders in epoxy, i.e., the binary locally resonant materials. Numerical simulations predict that the subfrequency gap also appears because of the high contrast of mass density and elastic constants of the soft rubber. The locally resonant mechanism is proved adequately and analyzed deeply. A simple quasi-one-dimensional mechanical analog model is introduced in order to understand the physical insight of the locally resonant mechanism more clearly. We discover for the first time the rule used to judge whether a resonant mode in the phononic crystals can result in a corresponding subfrequency gap or not.

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