A double porosity material for low frequency sound absorption

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ABSTRACT

This work designs a double porosity material (DPM) composed of two types of pores, i.e., the micro-pore from the porous layer and the meso-pore made by the labyrinthine channel. The both loss mechanisms of two different pores are combined to explore the low frequency sound absorption. All theoretical, numerical and experimental results show that the DPM possesses much lower frequency sound absorption than that of homogenous porous material (HPM) under the same thickness. Both the pressure and particle velocity distributions reveal that the sound absorption peaks are induced by the resonances of the labyrinthine channel and the hybrid resonance between the porous layer and labyrinthine channel respectively. Moreover, unconventional features such as negative bulk modulus and slow sound speed are observed around the resonant frequencies of the DPM. Finally, the absorption tailoring of the DPM with different strategies is investigated.

1. Introduction

Effective absorption of low frequency noise has been an important issue for decades, and it has attracted considerable concerns from scientists in circles of physics and engineering. Using a homogenous porous material (HPM) is an effective approach for sound absorption [1]. However, the thickness of the HPM with rigid backing requires about a quarter of the sound wavelength to realize efficient absorption, which results in the HPM being too thick if the low-frequency sound elimination is to be a target. Many researchers attempted to use multilayer structures to overcome the thickness constraint [2,3]. Nevertheless, owing to lack of subwavelength resonance, such a strategy cannot improve sound absorption significantly at frequency domain below the quarter-wavelength resonance (QWR).

In recent decades, introducing heterogeneities into the porous layer provides a promising direction for achieving efficient absorption below the frequency of the quarter-wavelength resonance (QWR). One possibility is using the concept of metaporous material (MPM), i.e., a porous matrix periodically embedded with inclusions [4–12]. Lagarrigue et al. [9] investigated a layer of MPM embedded with slotted cylinder. They confirmed that the MPM possesses quasi-perfect sound absorption with a thickness only 1/10 of the operation wavelength. Boutin and Becot [4] presented a theoretical investigation of MPMS containing various inclusions in the form of impervious spheres, Helmholtz resonators, or folded quarter-wavelength resonators, and the former two were demonstrated experimentally to be efficient for sound absorption at frequencies below 500 Hz. Another alternative choice is the double porosity materials (DPM), which consists of two interconnected pores of different characteristic size [13–18]. Boutin et al. [13] established an integrated theory to describe this kind of materials by the homogenization method. Sgard et al. [15] gave an experimental demonstration of a DPM in the form of perforation porous materials. They showed that good absorption performance at low frequencies can be acquired due to the acoustic coupling between two pore systems. Followed by these pioneer works, Ren et al. [17] theoretically and numerically investigated a DPM by introducing the micro-slit structures into a meso-slit matrix. Sound absorption enhancement is obtained in the frequency range of 1000–6000 Hz. Recently, Xin et al. [18] further considered the influence of pressure diffusion and proposed a multi-scale theoretical approach for the slit-perforated DPM.

The labyrinthine structure in acoustic metamaterial has been demonstrated to have powerful impedance manipulation [19–21] and tunability [22,23]. It has been widely introduced into sound absorbers composed of deep-subwavelength Helmholtz resonators [24–26] or Fabry–Pérot channels [27,28]. Inspired by these previous studies, there are two main motivations of this work: (1) to design a DPM by synthesizing advantages of the labyrinthine structure and porous layer and (2) further improve low-frequency sound absorption of the DPM under a thin layer.

The organization of the paper is as follows: Section 2 firstly shows the model of the designed DPM, and then a theoretical homogenization method [29] is introduced to predict sound absorption of the DPM. For
validation purpose, the numerical and experimental methods are also outlined. Section 3 investigates the absorption performance of the DPM. The underlying mechanisms are revealed by the physical field patterns and effective parameters. Moreover, the absorption tailoring of the DPM with different strategies is investigated. Finally, conclusion remarks are drawn in Section 4.

2. Models and methods

2.1. The models of the DPM

Fig. 1(a) shows diagrams of the conventional HPM and the designed DPM. Different from the conventional HPM, the DPM consists of alternately parallel arranged porous layers and labyrinthine channels along the y direction. The DPM unit has sizes of \( L \) and \( H \) in the \( y \) and \( z \) directions, respectively. The incident plane wave is along the \( -z \) direction. The DPM is infinite and invariant along the \( x \) direction, thus the DPM can be simplified as a two-dimensional model in the \( yz \) plane. Fig. 1(b) illustrates the cross-section diagram of the DPM unit in the \( yz \) plane. The width of the porous part is \( y_1 \) along the \( y \) axis. The labyrinthine channel has six layers (i.e., the folding number \( n = 6 \)) partitioned by rigid slabs (thickness \( b_0 \)). The width of each channel is uniform of \( w_i = (H - n \times b_0)/n \).

2.2. Theoretical method

Generally, the absorption coefficient \( \alpha \) of the DPM with rigid backing is given by

\[
\alpha = 1 - \frac{1 - Z_0/c_0}{1 + Z_0/c_0} \tag{1}
\]

where \( Z_0 = \rho_0 c_0 \) is the characteristic acoustic impedance of the air, \( \rho_0 \) and \( c_0 \) are the density and sound speed of the air, respectively. \( Z_0 \) is the surface acoustic impedance of the DPM, which takes the form as follows

\[
Z_0 = -j\sqrt{\rho_{DPM} \Phi_{DPM}} \cot(\omega H/c_{DPM}) \tag{2}
\]

where \( \omega \) is the angular frequency, \( j = \sqrt{-1} \), \( c_{DPM} = (\Phi_{DPM}/\rho_{DPM}) \) is the effective sound speed, \( \rho_{DPM} \) and \( \Phi_{DPM} \) are the effective density and bulk modulus of the DPM, respectively. According to the homogenization theory with the pressure diffusion effect ignored [29], they are

\[
\frac{1}{\rho_{DPM}} = \frac{\Phi_p}{\rho_p}, \tag{3}
\]

and

\[
\frac{1}{\Phi_{DPM}} = \frac{\Phi_{DPM}}{\rho_{DPM}} \tag{4}
\]

where \( \Phi_p = (H y_1)/(HL) \), \( \rho_p \) and \( B_p \) are the filling fraction, effective density and bulk modulus of the porous material, \( \Phi_{DPM} = (L w_i)/(LH) \), \( B_L \) and \( k_L \) are the filling fraction, effective bulk modulus and wavenumber of the air in the labyrinthine channel, respectively. \( L \) is the total length of the labyrinthine channel. Based on Johnson-Champoux-Allard (JCA) model [1], the effective density \( \rho_p \) and bulk modulus \( B_p \) of the porous layer are given by (with time convention \( \varepsilon^{-j\omega t} \))

\[
\begin{align*}
\frac{\eta}{-j\omega \rho_p} &= \frac{k_o}{\sqrt{1 - \frac{\omega^2}{\omega_v^2} \left( \frac{\lambda}{\lambda'} \right)^2}}\frac{\omega_v}{\omega_v^2} \sqrt{\frac{\Phi_p}{\Phi_{DPM}}}, \tag{5}
\end{align*}
\]

and

\[
B_p = \frac{\rho_0 y_0}{\eta + \|y - 1\|\omega c_v / \kappa \sqrt{1 - \frac{\omega^2}{\omega_v^2} \left( \frac{\lambda}{\lambda'} \right)^2}} \tag{6}
\]

where

\[
k' = \frac{k'}{k} = \frac{1}{\sqrt{1 - \frac{\omega^2}{\omega_v^2} \left( \frac{\lambda}{\lambda'} \right)^2}} - \frac{\omega^2}{\omega_v^2} \tag{7}
\]

and

\[
\omega_v = \frac{\Phi_p}{\rho_p k_0 \alpha_{\omega_v}}, \omega_v = \frac{\Phi_p}{C_v k_0}, \delta_v = \sqrt{\frac{\eta}{\rho_p \alpha_{\omega_v}}}, \delta_v = \sqrt{\frac{\kappa}{C_v \alpha_{\omega_v}}} \tag{8}
\]

Here, \( \omega_v \) is the characteristic frequency for the transition from viscous to inertial regimes, while \( \omega_v \) is the characteristic frequency for the transition from isothermal to adiabatic transformation. \( \delta_v \) and \( \delta_v^* \) are viscous and thermal skin-depths at the characteristic frequencies \( \omega_v \) and \( \omega_v \), while \( \kappa, C_v \) and \( \eta \) are the heat conductivity, isobaric heat capacity and dynamic viscosity coefficient of the air. Five parameters such as the porosity \( \Phi \), the flow resistivity \( \sigma \), the tortuosity \( \alpha_{\omega_v} \), the viscous length \( \Lambda \) and the thermal characteristic length \( \lambda' \) are used for describing properties of the porous material. \( k_0 = \eta/\sigma \) is the visco-static Darcy permeability, \( k_0^* \) is the thermostatic permeability.

Considering viscous and thermal losses in the narrow labyrinthine channel, the effective bulk modulus \( B_L \) and effective wavenumber \( k_L \) of the air in the channel are

\[
B_L = \frac{p_0}{\gamma - (y - 1)F(w_i)} \tag{9}
\]
\[ k_L = \frac{\omega}{\sqrt{\beta_z/\rho_L}} \]  \tag{10} \]

where \( \delta_L = \sqrt{x/\omega C_p} \) is the thermal skin-depths, \( F(X) = 1 - \tanh(X)/X \) is a form-function and \( \rho_L \) is the effective density, expressed by

\[ \rho_L = \frac{\rho_0}{F(\sqrt{x/\omega C_p})} \]  \tag{11} \]

where \( \delta_v = \sqrt{\eta/\omega \rho_0} \) is the viscous skin-depths.

2.3. Simulation and experiment method

To validate the above theoretical method, numerical simulation and experiment are conducted, respectively. Full-wave simulation is performed by the Acoustic-Thermoacoustic Interaction and Frequency domain module of commercial finite element (FE) software COMSOL Multiphysics, as shown in Fig. 2. The background acoustic field and the porous part use the Pressure Acoustic module where the Helmholtz equation is solved. The porous material is described by the Johnson-Champoux-Allard (JCA) model in Poroacoustics Domain. The labyrinthine channel uses the Thermoacoustics module where the Navier-Stokes equation, the continuity equation, and the energy equation are coupled and solved. The black solid line denotes the sound-hard boundary, and the dash-dotted lines are the periodic boundaries, respectively. The dashed line between the Poroacoustics Domain (red region) and the Thermoacoustics Domain (purple region) represents the coupled interface between the Pressure Acoustic module and the Thermoacoustics module.

The experiment is carried out by the two-microphone method [30] in an acoustic impedance tube with square cross-section, the side length of the impedance tube is \( d = 60 \) mm, as illustrated in Fig. 3. The locations of the microphones and the sample are set as \( x_1 = 200 \) mm and \( x_2 = 150 \) mm, respectively.

![Fig. 2. Illustration for the FE model of one unite cell.](image)

![Fig. 3. The sketch of the experimental set-up.](image)

3. Results and discussions

3.1. Sound absorption performance of the DPM

Fig. 4(a) compares the absorption spectra of the DPM sample obtained by the theory (blue dashed line), simulation (red solid line) and experiment (black circles), respectively. For comparison, the absorption spectrum of the conventional HPM of the same thickness is also plotted (see the pink dotted line from theory and the green circles from experiment). In the present computation, the parameters of air include \( c_0 = 343 \) m/s, \( \rho_0 = 1.21 \) kg/m\(^3\), \( \kappa = 0.026 \) W/(m·K), \( C_p = 1004 \) J/(kg·K), \( \eta = 1.81 \times 10^{-5} \) Pa·s and \( \beta_0 = 1.013 \times 10^5 \) Pa.

The sketch of the sample is illustrated in Fig. 4(b) and the sample photo is shown in Fig. 4(c). Here, the porous part is melamine foam with acoustical parameters listed in Table 1. The frame is fabricated by resin using three-dimensional printing. The structural parameters of the sample are presented in Table 2. From Fig. 4(a), it can be seen that there are three quasi-perfect absorption peaks below the QWR (at 2225 Hz) of the HPM, and significantly improves the low frequency absorption of the HPM. These absorption peaks appear at 451 Hz (443 Hz and 456 Hz), 1350 Hz (1334 Hz and 1416 Hz) and 2018 Hz (1975 Hz and 1896 Hz) with corresponding peak values of 100% (100% and 98.9%), 98.9% (98.3% and 99.2%) and 98.6% (98.4% and 99.8%) in the theory (simulation and experiment), respectively. The results of the theory, simulation and experiment show good coincidence. However, some discrepancies still exist. We attribute that the mismatch is induced by two main reasons. One is the uncertainty of sample fabrication, the other is that the porous material may be squeezed weakly when placed in the 3D-printed frame. Therefore the material parameters may be slightly changed. The overall thickness of the DPM, \( H = 30 \) mm, is only 1/25.4 of the sound wavelength at 451 Hz, which shows an obvious subwavelength thickness. The overall thickness of the DPM is also smaller than the previously reported DPMs [15–18].

Table 1

<table>
<thead>
<tr>
<th>Acoustical parameters of the melamine foam.</th>
<th>( \phi )</th>
<th>( \tau_m )</th>
<th>( \Lambda )</th>
<th>( \lambda' )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.95</td>
<td>1.42</td>
<td>180um</td>
<td>360um</td>
<td>8900 N s m(^{-4})</td>
</tr>
</tbody>
</table>
is mainly trapped in the porous layer, while partial acoustic energy exists in the labyrinthine channel. The DPM shows a hybrid resonance mode and the radiation impedance (induced by the second item of the right part in Eq. (4)) from labyrinthine channel shifts the QWR frequency of the porous layer to a hybrid resonance frequency (at 1975 Hz). The acoustic interaction between porous layer and labyrinthine channel, i.e., the hybrid resonance mode, leads to the absorption enhancement (compared to HPM) around the peak frequency.

### 3.2.2. Effective parameters of the DPM

To get further insight into the absorption performance, the effective parameters of the DPM are investigated. A retrieval method [31] in simulation is employed to validate the correctness of the effective parameters obtained by the theoretical model. As illustrated in Fig. 6, the pressure transmission coefficient $T$ and reflection coefficient $R$ can be calculated by removing the rigid backing of the DPM. Then the acoustic impedance $z_{DPM}$ and the refractive index $n_{DPM}$ take the following form

$$z_{DPM} = \frac{r}{1 - 2R + R^2 - T^2}, \quad n_{DPM} = \frac{-\log(x) + 2\pi}{kH}$$

where

$$r = \frac{(R^2 - T^2 - 1)^2 - 4R^2}{2T}$$

For the DPM without periodic distributions along the wave propagation direction, the branch number $l$ is zero. The wave vector $k$ takes the form $k = \omega/c_0$. For the two roots of $r$ in Eq. (13), the one with positive real part of $z_{DPM}$ should be chosen. Then the retrieved effective sound speed $c_{eff}$ and bulk modulus $B_{eff}$ are computed by

$$c_{DPM} = \frac{c_0}{n_{DPM}}$$

$$B_{DPM} = \frac{z_{DPM}}{c_{DPM}}$$

Fig. 7(a) and (b) depict the effective parameters of the DPM, here, the bulk modulus and sound speed are normalized by $\gamma P_0/\phi$ and $c_0$. Both theoretical and retrieved results show a good agreement. For comparison, the effective parameters of the HPM are also plotted. From Fig. 7(a), one can see that, around the resonant frequency, the bulk modulus of the DPM shows two drops into negative immediately followed by rapid jump around the first two resonant frequencies. This phenomenon is also consistent with the previous study on the porous media with inner resonators [4]. During the drop of the bulk modulus, it is observed that the sound speed decreases dramatically [see Fig. 7(b)], which leads to strong velocity dispersion. This velocity dispersion leads to an enhancement of sound absorption compared to the HPM within the frequency domain of the negative bulk modulus.

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**Table 2**

<table>
<thead>
<tr>
<th>$L$ (mm)</th>
<th>$H$ (mm)</th>
<th>$y_1$</th>
<th>$b_0$</th>
<th>$w_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>30</td>
<td>27</td>
<td>1.5</td>
<td>3.5</td>
</tr>
</tbody>
</table>

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Fig. 6. Schematic of the simulation model for retrieving effective parameters of the DPM. $I$ denotes the incident plane wave, $R$ and $T$ are the pressure reflection and transmission coefficients. The top and bottom are rigid boundaries, while the left and right boundaries are terminated with infinite elements.

Fig. 5. The normalized acoustic pressure field and velocity patterns (black arrows) at frequencies of (a) the first, (b) the second and (c) the third absorption peaks. $P$ is the total acoustic pressure and $P_i$ the pressure of the incident wave.

### 3.2. Absorption mechanism

#### 3.2.1. Pressure and velocity distributions

The labyrinthine channel can be regarded as a curled Fabry–Pérot (FP) channel. The resonance frequency is $f_{1m}^DPM = (2m - 1)c_L/4L_{eff}$ [28], where $L_{eff}$ is the total length of the channel, $c_L = \omega/\xi$, is the effective speed of sound in the labyrinthine channel, and $m$ is a positive integer ($m = 1, 2, 3, \ldots$). The first two absorption peaks of the DPM are mainly dominated by the fundamental resonance mode (i.e., $m = 1$, the corresponding FP resonance frequency $f_{11}^DPM = 468$ Hz) and the second-order resonance mode (i.e., $m = 2$, the corresponding FP resonance frequency $f_{22}^DPM = 1404$ Hz) of the labyrinthine channel, respectively. Fig. 5(a) and (b) further illustrate the distributions of the normalized pressure and particle velocity at first two absorption peak frequencies. For the first absorption peak frequency, one can see that the air pressure in the labyrinthine channel acquires the maximum at the end of the channel. While at the second absorption peak frequency, there exist two locations acquiring the maximum acoustic pressure in the labyrinthine channel, the air therein is divided into two regions with out-of-phase vibration, which is denoted by the arrow direction. The pressure and particle velocity patterns further demonstrate that the first two absorption peaks are mainly determined by the resonances of the labyrinthine channel. The lower frequencies of the first two resonant modes of the DPM than those predicted by the FP resonance stem from the weak acoustic coupling of porous layer and the labyrinthine channel. At the third absorption peak frequency (see Fig. 5(c)), the acoustic energy
3.3. Absorption tailoring

3.3.1. Filling fraction of the porous layer

In the above discussions, it is demonstrated that the DPM possesses multiple absorption peaks at frequency domain below the QWR of the HPM. In practical application, one need to adjust the absorption peaks to the desired frequency for optimal noise cancellation. For our DPM, one important strategy for tuning the absorption peaks is to change the filling fraction $\Phi$ of the DPM. Fig. 8 illustrates the absorption spectra of the DPM varying with $\Phi$. Here, the filling fraction $\Phi$ ranges from 40% to 60% with $y_1$ increased from 24 mm to 36 mm, while other parameters are identical to those listed in Table 1 and Table 2. From Fig. 8, one can see that the location of first (second) absorption peak can be tuned from 424 Hz (1268 Hz) to 668 Hz (1960 Hz) with quasi-total absorption acquired. Because the total length of the labyrinthine channel is shortened with $\Phi$ increased, the first-two absorption peaks move to higher frequency ranges. Moreover, it is noted that the third absorption peak also moves to higher frequency range slightly with $\Phi$ increased, and this is induced by different radiation impedance from the labyrinthine channel to the porous layer.

3.3.2. Multiple channels in parallel

Instead of using DPMs with different filling fraction of the porous layer, we further adopt two parallel labyrinthine channels along the $z$ axis to tailor the low frequency absorption peaks. Fig. 9(a) shows the cross-section of the DPM unit with two parallel labyrinthine channels of $n=4$ (denoted by C1) and $n=2$ (denoted by C2). Other dimensions of the structure and the acoustical parameters of the porous layer are also kept the same as those in Tables 1 and 2. Fig. 9(b) depicts the absorption spectra obtained by simulation and experiment. Four quasi-perfect absorption peaks exhibit at 609 Hz (646 Hz), 1196 Hz (1160 Hz), 1752 Hz (1660 Hz) and 2128 Hz (2060 Hz) in the simulation result (experiment), respectively. Compared with the previous single labyrinthine channel cases, the DPM with two parallel labyrinthine channels possesses richer resonance modes and it provides an approach to realize efficient absorption for more desired frequencies simultaneously.

4. Conclusions

In summary, we theoretically, numerically and experimentally investigate the low frequency absorption of the DPM with labyrinthine channels. It is shown that the designed DPM enhances obviously the sound absorption below the QWR of the conventional HPM. The ratio of the thickness of the DPM to the working wavelength can be less than 1/25, which is smaller than the previous DPMs [15–18]. The sound pressure and velocity distributions reveal that the enhancement of sound absorption is induced by the resonance of the labyrinthine channel and the hybrid resonance between the porous layer and the labyrinthine channel. Further, it reveals that the strong velocity dispersion is favorable for acoustic absorption. In contrast to HPM, the DPM also possesses more design freedoms to tailor the sound absorption spectrum at desired frequency domain. If wider bandwidth of the absorption is to be a target, one can use more unit cells with different structural and material parameters, or benefit from an optimization process.
CRediT authorship contribution statement

Honggang Zhao: Conceptualization, Writing - review & editing, Funding acquisition.
Yang Wang: Writing - review & editing, Software, Formal analysis.
Dianlong Yu: Validation, Funding acquisition.
Haibin Yang: Methodology.
Jie Zhong: Software.
Fei Wu: Visualization.
Jihong Wen: Supervision, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References