Coherent coupling based meta-structures for high acoustic absorption at 220–500 Hz frequency

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1. Introduction

Acoustic meta-structure with high absorption and board-band within low frequency range has been extensively investigated [1–6]. However, several aspects remain to be described, given the majority of the previous studies only achieved a single prefect sharp absorption peak [7,8] or absorption within narrow band [9–23]. Coherent coupling between weak Helmholtz Resonators (HR) has recently been proposed to achieve both high efficiency and broad-band sound absorption [25]. In this method, many arrays of HRs of different sizes were assembled into a unit cell. Singly, each HR was an elementary resonator in the unit cell of meta-structure [26,27], have been used to improve the absorption in the desired frequency range. Although these methods achieved the desired absorption performance, they required thick unit cell (200 mm).

2. Structures and methods

In order to understand the basis behind the high absorption in the low frequency range of 220 Hz–300 Hz for the novel meta-structures, we first describe the features of coherent coupling between weak resonators. As previously reported [7], the absorption coefficient of a neck-embedded HR is expressed as follows;

\[ Z = Z_A + Z_C + Z_D \]

\[ Z = \frac{A}{S_0} \left[ -\rho_0 c_0 \sqrt{2j \sin(k \alpha \eta_0 / 2)} \left( /c_0^2 - (\gamma - 1) /c_0^2 \right) \right] + \frac{A}{S_0} \left[ j2\omega \rho_0 c_0 /c_0^2 - \frac{j2\omega \rho_0 c_0 /c_0^2}{l_0} \cot(k l_0) \right] \]

Where \( \rho_0 \) is the static air density, \( c_0 \) is the speed of sound, \( c_1 \) and \( c_2 \) are the air density, sound speed and wave number inside the cavity, respectively. \( A \) is the front surface area of the unit cell, \( S_0 \) is the cross sectional area of the aperture, \( S_0 \) is the area of the cavity in the incidence plane whereas \( L \) is the length of the embedded aperture (other parameters are shown in the supplementary information SI Note 1). Therefore, the absorption coefficient of a single HR at normal incidence angle of sound can be expressed as follows;

\[ \chi = 1 - \frac{|Z - \rho_0 c_0|}{|Z + \rho_0 c_0|} \]

A single HR is an elementary resonator in the unit cell of meta-structures. And it only achieves a single absorption peak by viscous and thermal effects [29,30]. To broaden the absorption range, many HRs of different sizes are arranged together to form a unit cell with a thickness of 50 mm and a surface area of 100mm x 100mm. All HRs work collectively to significantly
enhance the absorption in the desired frequency range through the coherent coupling effect [25]. The corresponding impedance of the whole structure can be expressed as:

$$Z = 1 / \sum_{n=1}^{n} Z_n^{-1}$$

(3)

This approach has however been described in previous literature [25], and its corresponding structure is shown in Fig. 1(d), marked type A.

Type A meta-structure comprises 25 elementary resonators with an embedded aperture and a back cavity. Although the cavities in ERs are regular and invariable, the sizes and thicknesses of apertures vary based on the desired absorption peaks. This was the originally proposed structure, however, the related mechanical and physical properties that enhance coherent coupling haven’t been fully explored. Herein, we chose two very distant peaks (generated by two typical ERs in type A structure). Separated them far from each other and then recomposed them together with a frequency step of 2 Hz. With the evidence shown in the previous literature [25] and a lot of study done by us, it was convinced that small enough frequency interval between neighbouring low peaks is a prerequisite to generate strong coupling and get high absorption level. The corresponding absorption coefficient of type A meta-structure revealed that the 25 peaks seemed to distribute out of order and some of them even overlapped in the full range. Indeed, the discrete peaks were low, therefore lots of similar peaks were employed to overlap where the coherent coupling was weak and the coupled absorption was low. Therefore, the raw peaks were necessary to be high enough.

Overlapping of peaks is useful but not indispensable. Indeed, the absorption coefficient at a fixed frequency results from the superposition of all absorption (by ERs), similar to the superposition of electromagnetic waves (see the SI Note 2). As such, we only need to locate peaks at the points where other peaks intersect. In this way, it is possible to decrease the total number of peaks while retaining a high enough coherent coupling effect.

For the reason why the superfluous peaks should be deleted, it is that the size and thickness of unit cell are strictly limited as mentioned above. And the target frequency range is extended from 300 Hz–500 Hz to 220 Hz–500 Hz under the same unit cell. If the previous 25 peaks were maintained, then a high absorption would not be obtained in the range of 220 Hz–300 Hz. Therefore, some of the peaks must be spared to the range of 220 Hz–300 Hz. Additionally, a more efficient strategy to couple the peaks must be explored because the peak value of an ER will rapidly decreases with respect to frequency.

Upon completing designing the suitable structures, the previous type A prototype was dropped since the distances between peaks produced by different ERs were too small and it could not effectively absorb waves within the target frequency range. As such, the cavity thickness was reduced by 1/5, based on the effect of front surface area $A$ in equation (1), as demonstrated to strongly affect the coupling effect. Similar findings have previously been reported [28]. By carefully varying the surface area, we found that increase in the quantity of peaks (i.e. increasing $A$) was negatively
correlated with the absorption coefficient (see SI Note 3). On the contrary, if the peak quantity was too small (i.e. decreasing $A$), then the coupling effect would weaken as the distances between peaks increased. Since in our case $A$ is fixed, the only way to lower the negative impact of large $A$ is by decreasing the terms in square brackets of equation (1).

The short structure, however, only yielded very low peaks because of the huge mismatch of acoustic impedance in the frequency range of 200 Hz-300 Hz. Therefore, we assembled a compact cavity (thickness $= 1/5$ of that in type A) to the above shortened structure by a thin plate, as shown in Fig. 1(b) (marked as type B). In addition, there is a short aperture on the plate for the connection of the upper cavity and the bottom cavity. The aperture is only 0.6mm thick to reduce the impedance. Consequently, the two cavities are arranged in series. Meanwhile, the lengths of the new apertures are 4/5 of those in type A. These changes ensure that the $Z_{is}$ decreased by 1/5 (see SI Note 4). That is the impedance of the new structure. Essentially, the front surface area of unit cell is decreased from 25$A_0$ ($A_0$ is the surface area of the bigger ER) to 20$A_0$. In addition, overlapping of peaks was applied at the lowest frequency point to further strengthen the coupling efficiency.

For the determination of ERs (depicted as peaks), two peaks were identified at either limits of the target frequency range, and other peaks were tailored at the points that peaks intersect with each other (the method of superposition, depicted in SI Note 2).

There are two different types of elementary resonator series in the unit cell which can be described as the bigger and the smaller series (the geometric parameters are listed in SI Note 6). The bigger series (quantity $= 16$) have absorption peaks in the range of 221 Hz-340 Hz whereas the smaller series (quantity $= 18$) have absorption peaks between 340 and 482 Hz. This is because at high frequency, the peaks of bigger series reach their optimal level, and thus the coherent coupling decreases, rather than enhances the total absorption.

Acoustic high-pass filter can also achieve similar results [24]. It mainly focuses on the introduction of a side branch aperture that is perpendicularly inserted to the original one. The configuration is shown in Fig. 2.

The size and thickness of the unit cell is the same as that of the type B meta-structure, but there is significant change in the elementary resonator. There is only one layer in normal direction (z direction) of the whole structure. Both the vertical aperture and the side branch are circular. The radius of the vertical aperture is 2.5 mm whereas that of the side branch is 2.4 mm (the detailed geometric parameters of all apertures are listed in SI Note 6). Position of the side branch in the vertical aperture determines both the peak positions and values. This relationship between the position of the side branch in the vertical aperture and both the peak positions and peak values is described by the impedance of the ER. It can be expressed as:

![Image](image_url)
Given the side branch divides the vertical aperture into two parts of different lengths, \( Z_{a1} + Z_{c1} \) refers to the absorption restrictions of the upper part, whereas \( Z_{a2} \) denotes the absorption restrictions of the bottom section. Because the bottom section is sealed, there is no radiation impedance and its relative end correction. \( Z_b \) is the impedance of the side branch \([24]\), expressed as follows:

\[
Z_b = \frac{\rho_0 c_0 k^2}{4\pi} + j\omega \left( \frac{\rho_0 (b + 0.7d_b)}{\pi d_b^2} \right)
\]  

(5)

Where \( b \) is the length of the side branch and \( d_b \) is the corresponding diameter. Essentially, because \( Z_b \) is highly dependent on the aperture length, it is easy to concurrently vary the peak positions and values. Also, the side branch \( Z_b \) is used to decrease of the restriction of the original aperture (without side branch), by lowering the impact of the magnification of front surface area under the same conditions.

### 3. Results and discussion

After design and construction, we simulated and calculated the absorption coefficients of the novel meta-structures. The simulation was performed using COMSOL Multiphysics™ Version 5.3 with a preset Acoustic-Thermoacoustic interaction module. Hard boundaries were imposed on all the interfaces between air and solid due to their large impedance mismatch. The sound was projected perpendicularly to the front surface of the structures at 1 Pa. For the first structure, the absorption coefficients of the discrete peaks in the range of 200 Hz–300 Hz are higher than 0.33, almost 1.2 times greater than those in type A (see SI Note 7), thus they are sufficient to support high coupling effect. We also experimentally measured the absorption coefficients of the two meta-structures using the impedance tube method compatible with ASTM C384-04 (2011) and ASTM E1050-12 (details shown in SI Note 5). The findings were summarized in Fig. 3. Here, the final absorption coefficient is greater than 0.8 within the frequency range of 221 Hz–482 Hz. The theoretical estimates differ with the simulated projection by less than 3%. It may stem from the approximation we made and the omission of the actual sizes of structures in calculation. Furthermore, the experimental result indicates that the meta-structures offers a broader frequency range with average absorption coefficient greater than 0.8 within 244 Hz to 540 Hz. The bigger deviation can be attributed to the fabrication error and the sound leaking problem. If we reasonably set the frequency range of target absorption level, the sound leaking problem significantly increases the absorption efficiency within 221 Hz to 290 Hz frequencies.

Further, the performance of the second structure was analyzed using a similar procedure as type B meta-structure (Fig. 4). The simulated absorption coefficient of the second meta-structure within 240–500 Hz is at least 0.8. Meanwhile, the calculated curve almost fits perfectly in the simulated curve. The small deviation can be attributed to the same reason as we concluded in the discussion of that in structure one. These two curves demonstrates that we successfully extended the lower limit of the valid frequency range from 290 Hz to 240 Hz. Overall, the acoustic absorption within 240 Hz to 290 Hz is significantly enhanced. Although the experimental measurement shows a somewhat bigger deviation from the simulation result, the difference is insignificant. The deviation mainly arises from sound leaking problem given the sample is a modification of two separate sections.

### 4. Conclusions

This research demonstrates the properties underlying the coherent coupling effect between Helmholtz resonators. In general, the sound incidence area is an important factor as it highly affects the coupling efficiency. To reduce the impact of sound incidence area, we devised two types of meta-structures while retaining the size of the unit cell. The new designs significantly improves the acoustic absorption within 220–300 Hz frequency range and extends the absorption range (221–500 Hz). Because the novel structures are easy to build, it will facilitate the design of meta-structures to diminish low frequency noise over a wide range.

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.apacoust.2021.108181.

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